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INTERNATIONAL ADVANCED LEVEL

MATHEMATICS/ FURTHER MATHEMATICS/ PURE MATHEMATICS

SAMPLE ASSESSMENT MATERIALS

Pearson Edexcel International Advanced Subsidiary in Mathematics (XMA11)
Pearson Edexcel International Advanced Subsidiary in Further Mathematics (XFM11)
Pearson Edexcel International Advanced Subsidiary in Pure Mathematics (XPM11)
Pearson Edexcel International Advanced Level in Mathematics (YMA11)
Pearson Edexcel International Advanced Level in Further Mathematics (YFM11)
Pearson Edexcel International Advanced Level in Pure Mathematics (YPM11)
First teaching September 2018
First examination from January 2019
First certification from August 2019 (International Advanced Subsidiary) and August 2020 (International Advanced Level)

Pearson

Edexcel, BTEC and LCCI qualifications

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Contents

Introduction	1
General marking guidance	3
Unit P1 – sample question paper and mark scheme	5
Unit P2 – sample question scheme and mark scheme	45
Unit P3 – sample question paper and mark scheme	87
Unit P4 – sample question paper and mark scheme	137
Unit FP1 – sample question paper and mark scheme	181
Unit FP2 – sample question paper and mark scheme	235
Unit FP3 – sample question paper and mark scheme	277
Unit M1 – sample question paper and mark scheme	323
Unit M2 – sample question paper and mark scheme	357
Unit M3 – sample question paper and mark scheme	399
Unit S1 – sample question paper and mark scheme	433
Unit S2 – sample question paper and mark scheme	473
Unit S3 – sample question paper and mark scheme	513
Unit D1 – sample question paper and mark scheme	557

Introduction

The Pearson Edexcel International Advanced Subsidiary in Mathematics, Further Mathematics and Pure Mathematics and the Pearson Edexcel International Advanced Level in Mathematics, Further Mathematics and Pure Mathematics are part of a suite of International Advanced Level qualifications offered by Pearson.

These sample assessment materials have been developed to support these qualifications and will be used as the benchmark to develop the assessment students will take.

For units P1, P2, P3, P4 and D1, the sample assessment materials have been formed using questions from different past papers from legacy qualifications, together with some new questions. For units FP1-FP3, M1-M3 and S1-S3, the sample assessment materials have been formed using whole past question papers from legacy qualifications.

The booklet '*Mathematical Formulae and Statistical Tables*' will be provided for use with these assessments and can be downloaded from our website, qualifications.pearson.com.

General marking guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked **unless** the candidate has replaced it with an alternative response.

Specific guidance for mathematics

1. These mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.
2. Abbreviations

   These are some of the traditional marking abbreviations that may appear in the mark schemes.

   - **bod** benefit of doubt
   - **ft** follow through
   - $\surd$ this symbol is used for correct ft
   - **cao** correct answer only
   - **cso** correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** ignore subsequent working
   - **awrt** answers which round to
   - **SC:** special case
   - **o.e.** or equivalent (and appropriate)
   - **d…** dependent or **dep**
   - **indep** independent
   - **dp** decimal places
   - **sf** significant figures
   - ***** The answer is printed on the paper or ag- answer given

- or d… The second mark is dependent on gaining the first mark

3. All M marks are follow through.

   All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

Sample Assessment Materials for first teaching September 2018
Time: 1 hour 30 minutes

Paper Reference
WMA11/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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1/1/1/1/1/

S 5 9 7 5 4 A 0 1 2 6

Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. Given that $y = 4x^3 - \dfrac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\dfrac{dy}{dx}$,

(3)

(b) $\int y \, dx$

(3)

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Question 1 continued

Q1

(Total for Question 1 is 6 marks)

Leave blank

2. (a) Given that $3^{-1.5} = a\sqrt{3}$ find the exact value of a

(2)

(b) Simplify fully $\dfrac{(2x^{\frac{1}{2}})^3}{4x^2}$

(3)

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Question 2 continued

Q2

(Total for Question 2 is 5 marks)

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3. Solve the simultaneous equations

$$y + 4x + 1 = 0$$

$$y^2 + 5x^2 + 2x = 0$$

(6)

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Question 3 continued

Q3

(Total for Question 3 is 6 marks)

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4. The straight line with equation $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = 2x^2 + 8x + 3$

Calculate the value of c

(5)

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Question 4 continued

Q4

(Total for Question 4 is 5 marks)

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5. (a) On the same axes, sketch the graphs of $y = x + 2$ and $y = x^2 - x - 6$ showing the coordinates of all points at which each graph crosses the coordinate axes.

(4)

(b) On your sketch, show, by shading, the region R defined by the inequalities

$$y < x + 2 \quad \textbf{and} \quad y > x^2 - x - 6$$

(1)

(c) Hence, or otherwise, find the set of values of x for which $x^2 - 2x - 8 < 0$

(3)

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Question 5 continued

Q5

(Total for Question 5 is 8 marks)

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6.

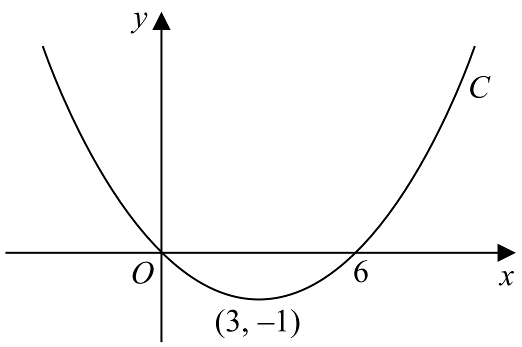

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$

The curve C passes through the origin and through (6, 0)

The curve C has a minimum at the point (3, –1)

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(2x)$

(3)

(b) $y = \mathrm{f}(x + p)$, where p is a constant and $0 < p < 3$

(4)

On each diagram show the coordinates of any points where the curve intersects the x-axis and of any minimum or maximum points.

Leave blank

Question 6 continued

Q6

(Total for Question 6 is 7 marks)

Leave blank

7. A curve with equation $y = \mathrm{f}(x)$ passes through the point (4, 25)

Given that

$$\mathrm{f}'(x) = \frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1, \qquad x > 0$$

find $\mathrm{f}(x)$, simplifying each term.

(5)

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Question 7 continued

Q7

(Total for Question 7 is 5 marks)

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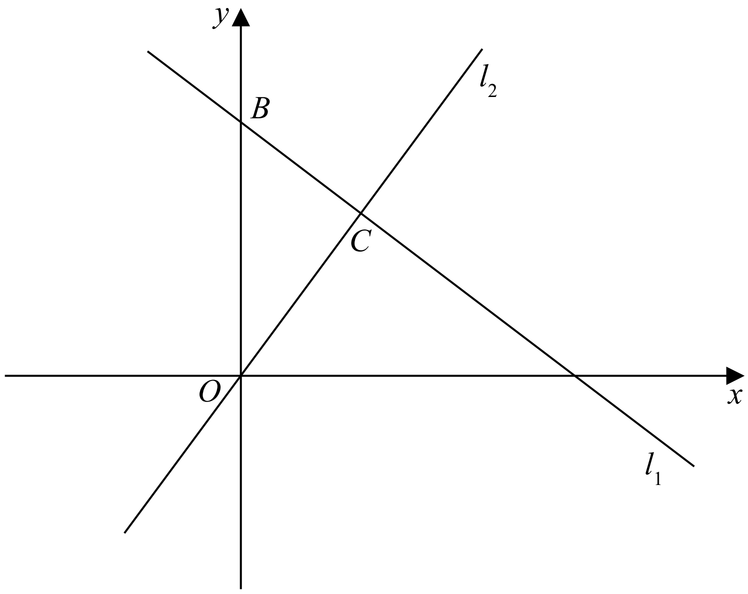

Figure 2

The line l_1, shown in Figure 2 has equation $2x + 3y = 26$

The line l_2 passes through the origin O and is perpendicular to l_1

(a) Find an equation for the line l_2

(4)

The line l_2 intersects the line l_1 at the point C. Line l_1 crosses the y-axis at the point B as shown in Figure 2.

(b) Find the area of triangle OBC. Give your answer in the form $\frac{a}{b}$, where a and b are integers to be found.

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

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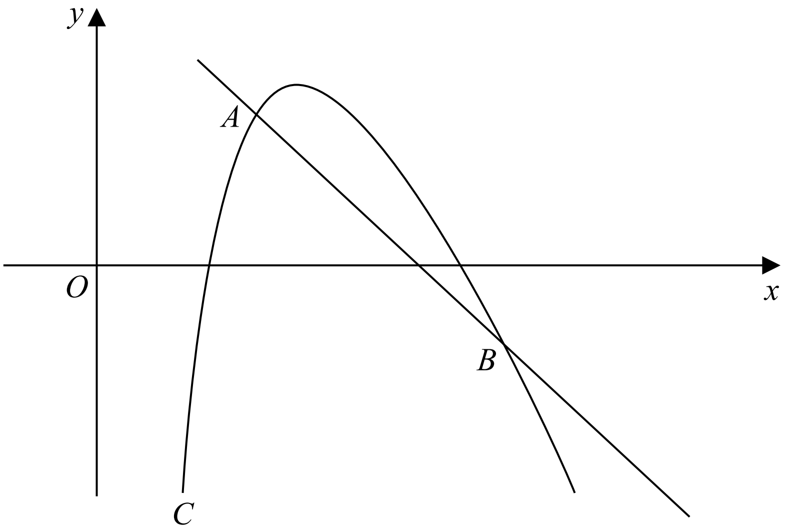

Figure 3

A sketch of part of the curve C with equation

$$y = 20 - 4x - \frac{18}{x}, \qquad x > 0$$

is shown in Figure 3.

Point A lies on C and has x coordinate equal to 2

(a) Show that the equation of the normal to C at A is $y = -2x + 7$.

(6)

The normal to C at A meets C again at the point B, as shown in Figure 3.

(b) Use algebra to find the coordinates of B.

(5)

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Question 9 continued

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Question 9 continued

Q9

(Total for Question 9 is 11 marks)

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10.

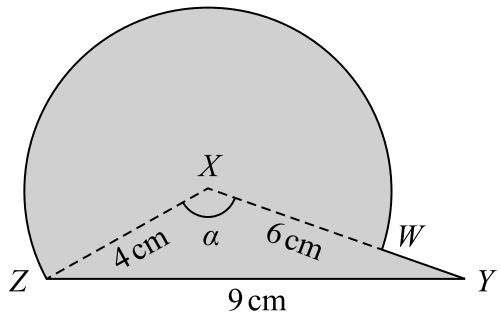

Figure 4

The triangle XYZ in Figure 4 has $XY = 6$ cm, $YZ = 9$ cm, $ZX = 4$ cm and angle $ZXY = \alpha$.

The point W lies on the line XY.

The circular arc ZW, in Figure 4, is a major arc of the circle with centre X and radius 4 cm.

(a) Show that, to 3 significant figures, $\alpha = 2.22$ radians.

(2)

(b) Find the area, in cm^2, of the major sector $XZWX$.

(3)

The region, shown shaded in Figure 4, is to be used as a design for a logo.

Calculate

(c) the area of the logo

(3)

(d) the perimeter of the logo.

(4)

Leave blank

Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

Pure Mathematics P1 Mark scheme

Question	Scheme	Marks
1(a)	$y = 4x^3 - \dfrac{5}{x^2}$	
	$x^n \to x^{n-1}$ e.g. sight of x^2 **or** x^{-3} **or** $\dfrac{1}{x^3}$	M1
	$3 \times 4x^2$ **or** $-5 \times -2x^{-3}$ (o.e.) (Ignore + c for this mark)	A1
	$12x^2 + \dfrac{10}{x^3}$ **or** $12x^2 + 10x^{-3}$ <u>all on one line</u> and no + c	A1
		(3)
(b)	$x^n \to x^{n+1}$ e.g. sight of x^4 **or** x^{-1} **or** $\dfrac{1}{x^1}$	M1
	Do <u>not</u> award for integrating their answer to part (a) $4\dfrac{x^4}{4}$ **or** $-5 \times \dfrac{x^{-1}}{-1}$	A1
	For fully correct and simplified answer with + c <u>all on one line.</u> Allow $\Rightarrow$ Allow $x^4 + 5 \times \dfrac{1}{x} + c$ $\Rightarrow$ Allow $1x^4$ for x^4	A1
		(3)
		(6 marks)

Question	Scheme		Marks
2(a)	$3^{-1.5} = \frac{1}{3\sqrt{3}} \quad \left(\frac{\times\sqrt{3}}{\times\sqrt{3}}\right)$		M1
	$= \frac{\sqrt{3}}{9}$ so $a = \frac{1}{9}$		A1
			(2)
	Alternative		
	$3^{-1.5} = a\sqrt{3} \Rightarrow a = \frac{3^{-1.5}}{3^{0.5}} = 3^{-1.5-0.5}$		M1
	$\Rightarrow a = 3^{-2} = \frac{1}{9}$		A1
(b)	$\left(2x^{\frac{1}{2}}\right)^3 = 2^3 x^{\frac{3}{2}}$	One correct power either 2^3 or $x^{\frac{3}{2}}$.	M1
	$\frac{8x^{\frac{3}{2}}}{4x^2} = 2x^{-\frac{1}{2}}$ **or** $\frac{2}{\sqrt{x}}$		dM1 A1
			(3)
			(5 marks)

Notes:

(a)

M1: Scored for a full attempt to write $3^{-1.5}$ in the form $a\sqrt{3}$ or, as an alternative, makes a the subject and attempts to combine the powers of 3

A1: For $a = \frac{1}{9}$ Note: A correct answer with no working scores full marks

(b)

M1: For an attempt to expand $\left(2x^{\frac{1}{2}}\right)^3$ Scored for one correct power either 2^3 or $x^{\frac{3}{2}}$.

$\left(2x^{\frac{1}{2}}\right) \times \left(2x^{\frac{1}{2}}\right) \times \left(2x^{\frac{1}{2}}\right)$ on its own is not sufficient for this mark.

dM1: For dividing their coefficients of x **and** subtracting their powers of x. Dependent upon the previous M1

A1: Correct answer $2x^{-\frac{1}{2}}$ **or** $\frac{2}{\sqrt{x}}$

Question	Scheme		Marks
3	$y = -4x - 1$ $\Rightarrow (-4x-1)^2 + 5x^2 + 2x = 0$	Attempts to makes y the subject of the linear equation and substitutes into the other equation.	M1
	$21x^2 + 10x + 1 = 0$	Correct 3 term quadratic	A1
	$(7x+1)(3x+1) = 0 \Rightarrow (x =) -\frac{1}{7}, -\frac{1}{3}$	dM1: Solves a **3 term** quadratic by the usual rules A1: $(x =)\ -\frac{1}{7}, -\frac{1}{3}$	dM1A1
	$y = -\frac{3}{7}, \frac{1}{3}$	M1: Substitutes to find at least one y value A1: $y = -\frac{3}{7}, \frac{1}{3}$	M1 A1
			(6)
	Alternative		
	$x = -\frac{1}{4}y - \frac{1}{4}$ $\Rightarrow y^2 + 5\left(-\frac{1}{4}y - \frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}y - \frac{1}{4}\right) = 0$	Attempts to makes x the subject of the linear equation and substitutes into the other equation.	M1
	$\frac{21}{16}y^2 + \frac{1}{8}y - \frac{3}{16} = 0$ $(21y^2 + 2y - 3 = 0)$	Correct 3 term quadratic	A1
	$(7y+3)(3y-1) = 0 \Rightarrow (y =) -\frac{3}{7}, \frac{1}{3}$	Solves a **3 term** quadratic	dM1
		$(y =) -\frac{3}{7}, \frac{1}{3}$	A1
	$x = -\frac{1}{7}, -\frac{1}{3}$	Substitutes to find at least one x value.	M1
		$x = -\frac{1}{7}, -\frac{1}{3}$	A1
			(6)
			(6 marks)

Question	Scheme	Marks
4	Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ o.e.	A1
	States that $b^2 - 4ac = 0$	dM1
	$4^2 - 4\times2\times(3 - c) = 0$ and so $c =$	dM1
	$c = 1$ **cso**	A1
		(5)
	Alternative 1A	
	Sets derivative "$4x + 8$" $= 4 \Rightarrow x =$	M1
	$x = -1$	A1
	Substitute $x = -1$ in $y = 2x^2 + 8x + 3$ ($\Rightarrow y = -3$)	dM1
	Substitute $x = -1$ and $y = -3$ in $y = 4x + c$ or into $(y + 3)=4(x + 1)$ and expand	dM1
	$c = 1$ or writing $y = 4x + 1$ **cso**	A1
		(5)
	Alternative 1B	
	Sets derivative "$4x+8$" $=4 \Rightarrow x =$,	M1
	$x = -1$	A1
	Substitute $x =$ -1 in $2x^2+8x+3 = 4x+c$	dM1
	Attempts to find value of c	dM1
	$c = 1$ or writing $y = 4x + 1$ **cso**	A1
		(5)
	Alternative 2	
	Sets $2x^2+8x+3=4x+\mathrm{c}$ and collects x terms together	M1
	Obtains $2x^2+4x+3-c=0$ or equivalent	A1
	States that $b^2-4ac=0$	dM1
	$4^2-4\times2\times(3-c)=0$ and so $c =$	dM1
	$c = 1$ **cso**	A1
		(5)
	Alternative 3	
	Sets $2x^2+8x+3=4x+\mathrm{c}$ and collects x terms together	M1
	Obtains $2x^2+4x+3-c=0$ or equivalent	A1
	Uses $2(x+1)^2-2+3-c=0$ or equivalent	dM1
	Writes $-2 + 3 - c = 0$	dM1
	So $c = 1$ **cso**	A1
		(5)
		(5 marks)

Question 4 *continued*
Notes:

Method 1A

M1: Attempts to solve their $\frac{dy}{dx} = 4$. They must reach $x = ...$ (Just differentiating is M0 A0).

A1: $x = -1$ (If this follows $\frac{dy}{dx} = 4x + 8$, then give M1 A1 by implication).

dM1: (Depends on previous M mark) Substitutes their x = -1 into f(x) or into "their f(x) from (b)" to find y.

dM1: (Depends on both previous M marks) Substitutes their x = -1 and their y = -3 values into y = $4x + c$ to find c or uses equation of line is (y + "3") = 4(x + "1") and rearranges to $y = mx+c$

A1: $c = 1$ or allow for $y = 4x + 1$ cso.

Method 1B

M1A1: Exactly as in Method 1A above.

dM1: (Depends on previous M mark) Substitutes **their** x = -1 into $2x^2 + 8x + 3 = 4x + c$

dM1: Attempts to find value of c then A1 as before.

Method 2

M1: Sets $2x^2 + 8x + 3 = 4x + \mathrm{c}$ and tries to collect x terms together.

A1: Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ or even $2x^2 + 4x = c - 3$. Allow "=0" to be missing on RHS.

dM1: Then use completion of square $2(x+1)^2 - 2 + 3 - c = 0$ (Allow $2(x+1)^2 - k + 3 - c = 0$) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square.

dM1: $-2 + 3 - c = 0$ AND leading to a solution for c (Allow $-1 + 3 - c = 0$) ($x = -1$ has been used)

A1: $c = 1$ **cso**

Method 3

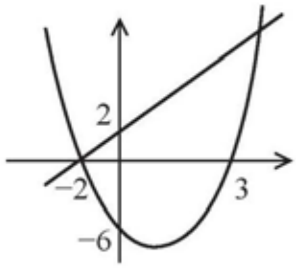
M1: Sets $2x^2 + 8x + 3 = 4x + \mathrm{c}$ and tries to collect x terms together. May be implied by $2x^2 + 8x + 3 - 4x \pm \mathrm{c}$ on one side.

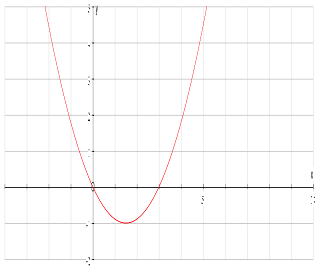
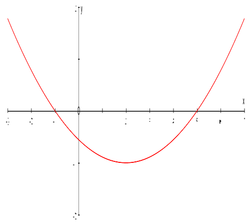
A1: Collects terms e.g. $2x^2 + 4x + 3 - c = 0$ or $-2x^2 - 4x - 3 + c = 0$ or $2x^2 + 4x + 3 = c$ even $2x^2 + 4x = c - 3$. Allow "=0" to be missing on RHS.

dM1: Then use completion of square $2(x+1)^2 - k + 3 - c = 0$ (Allow $2(x+1)^2 - k + 3 - c = 0$) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square.

dM1: $-2 + 3 - c = 0$ AND leading to a solution for c (Allow $-1 + 3 - c = 0$) ($x = -1$ has been used)

A1: $c = 1$ cso

<table>
<tr><th>Question</th><th colspan="2"></th><th>Marks</th></tr>
<tr><td rowspan="5">5(a)</td><td rowspan="4"></td><td>Straight line, positive gradient positive intercept</td><td>B1</td></tr>
<tr><td>Curve ‘U’ shape anywhere</td><td>B1</td></tr>
<tr><td>Correct y intercepts 2, -6</td><td>B1</td></tr>
<tr><td>Correct x-intercepts of -2 and 3 with intersection shown at $(-2, 0)$</td><td>B1</td></tr>
<tr><td colspan="2"></td><td>(4)</td></tr>
<tr><td rowspan="2">(b)</td><td colspan="2">Finite region between line and curve shaded</td><td>B1</td></tr>
<tr><td colspan="2"></td><td>(1)</td></tr>
<tr><td rowspan="4">(c)</td><td colspan="2">$(x^2 - x - 6 < x + 2) \Rightarrow x^2 - 2x - 8 < 0$</td><td></td></tr>
<tr><td colspan="2">$(x - 4)(x + 2) < 0 \Rightarrow$ Line and curve intersect at $x = 4$ and $x = -2$</td><td>M1 A1</td></tr>
<tr><td colspan="2">$-2 < x < 4$</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(3)</td></tr>
<tr><td colspan="4">(8 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">(a) As scheme.</td></tr>
<tr><td colspan="4">(b) As scheme.</td></tr>
<tr><td colspan="4">(c)
M1: For a valid attempt to solve the equation $x^2 - 2x - 8 = 0$
A1: For $x = 4$ and $x = -2$
A1: $-2 < x < 4$</td></tr>
</table>

Question	Scheme		Marks
6(a)		Shape through (0, 0)	B1
		(3, 0)	B1
		(1.5, −1)	B1
			(3)
(b)		Shape , <u>not</u> through (0, 0)	B1
		Minimum in 4^{th} quadrant	B1
		$(-p, 0)$ and $(6 - p, 0)$	B1
		$(3 - p, -1)$	B1
			(4)
			(7 marks)

Notes:

(a)

B1: U shaped parabola through origin.

B1: (3,0) stated or 3 labelled on x - axis (even (0,3) on x - axis).

B1: (1.5, -1) or equivalent e.g. (3/2, -1) labelled or stated and matching minimum point on the graph.

(b)

B1: Is for any translated curve to left or right or up or down not through origin

B1: Is for minimum in 4^{th} quadrant and x intercepts to left and right of y axis (i.e. correct position).

B1: Coordinates stated or shown on x axis (Allow $(0 - p, 0)$ instead of $(-p, 0)$)

B1: Coordinates stated.

Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case none of the curves should go through the origin for M1 and all minima should be in fourth quadrant and **all x intercepts need to be to left and right of y axis for A1**

Question	Scheme	Marks
7	$\mathrm{f}(x) = \int\left(\frac{3}{8}x^2 - 10x^{-\frac{1}{2}} + 1\right)\mathrm{d}x$	
	$x^n \to x^{n+1} \Rightarrow \mathrm{f}(x) = \frac{3}{8}\times\frac{x^3}{3} - 10\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + x(+c)$	M1 A1 A1
	Substitute $x = 4, y = 25 \Rightarrow 25 = 8 - 40 + 4 + c$ $\Rightarrow c =$	M1
	$\mathrm{f}(x) = \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53$	A1
		(5)
		(5 marks)

Notes:

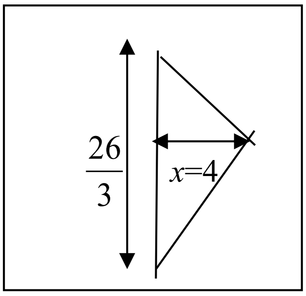
M1: Attempt to integrate $x^n \to x^{n+1}$

A1: Term in x^3 **or** term in $x^{\frac{1}{2}}$ correct, coefficient need not be simplified, no need for $+x$ nor +c

A1: ALL three terms correct, coefficients need not be simplified, no need for + c

M1: For using $x = 4$, $y = 25$ in their f(x) to form a linear equation in c and attempt to find c

A1: $= \frac{x^3}{8} - 20x^{\frac{1}{2}} + x + 53$ cao (all coefficients and powers must be simplified to give this answer- do not need a left hand side and if there is one it may be f(x) or y). Need full expression with 53. These marks need to be scored in part (a).

Question	Scheme	Marks
8(a)	$2x + 3y = 26 \Rightarrow 3y = 26 \pm 2x$ and attempt to find m from $y = mx + c$	M1
	$\left(\Rightarrow y = \frac{26}{3} - \frac{2}{3}x\right)$ so gradient = $-\frac{2}{3}$	A1
	Gradient of perpendicular $= \frac{-1}{\text{their gradient}}$ $\left(=\frac{3}{2}\right)$	M1
	Line goes through (0, 0) so $y = \frac{3}{2}x$	A1
		(4)
(b)	Solves their $y = \frac{3}{2}x$ with their $2x + 3y = 26$ to form equation in x or in y	M1
	Solves their equation in x or in y to obtain $x =$ **or** $y =$	dM1
	$x = 4$ or any equivalent e.g. $\frac{156}{39}$ **or** $y = 6$ o.a.e	A1
	$B = (0, \frac{26}{3})$ used or stated in (b)	B1
	 Area $= \frac{1}{2} \times "4" \times \frac{"26"}{3}$	dM1
	$= \frac{52}{3}$ (o.e. with integer numerator and denominator)	A1
		(6)
		(10 marks)

Notes:

(a)

M1: Complete method for finding gradient. (This may be implied by later correct answers.) e.g.
Rearranges $2x + 3y = 26 \Rightarrow y = mx + c$ so $m =$
Or finds coordinates of two points on line and finds gradient e.g.
$(13, 0)$ *and* $(1, 8)$ *so* $m = \frac{8-0}{1-13}$

A1: States or implies that gradient $= -\frac{2}{3}$ condone $= -\frac{2}{3}x$ if they continue correctly. Ignore errors in constant term in straight line equation.

M1: Uses $m_1 \times m_2 = -1$ to find the gradient of l_2. This can be implied by the use of $\frac{-1}{\text{their gradient}}$

A1: $y = \frac{3}{2}x$ or $2y - 3x = 0$ Allow $y = \frac{3}{2}x + 0$ Also accept $2y = 3x$, $y = \frac{39}{26}x$ or even $y - 0 = \frac{3}{2}(x - 0)$ and isw.

Question 8 notes *continued*

(b)

M1: Eliminates variable between their $y=\frac{3}{2}x$ and their (possibly rearranged) $2x+3y=26$ to form an equation in x or y. (They may have made errors in their rearrangement).

dM1: (Depends on previous M mark) Attempts to solve their equation to find the value of x or y

A1: $x = 4$ or equivalent or $y = 6$ or equivalent

B1: y coordinate of B is $\frac{26}{3}$ (stated or implied) - isw if written as $(\frac{26}{3}, 0)$.

Must be used or stated in (b)

dM1: (Depends on previous M mark) Complete method to find area of triangle OBC (using their values of x and/or y at point C and their $\frac{26}{3}$)

A1: Cao $\frac{52}{3}$ or $\frac{104}{6}$ or $\frac{1352}{78}$ o.e

Alternative 1

Uses the area of a triangle formula ½×OB ×(x coordinate of C)

Alternative methods: Several Methods are shown below. The only mark which differs from Alternative 1 is the last M mark and its use in each case is described below:

Alternative 2

In 8(b) using $\frac{1}{2}\times BC\times OC$

dM1: Uses the area of a triangle formula ½×BC ×OC Also finds OC (=√52) and BC= ($\frac{4}{3}\sqrt{13}$)

Alternative 3

In 8(b) using $\frac{1}{2}\begin{vmatrix}0 & 4 & 0 & 0\\ 0 & 6 & \frac{26}{3} & 0\end{vmatrix}$

dM1: States the area of a triangle formula $\frac{1}{2}\begin{vmatrix}0 & 4 & 0 & 0\\ 0 & 6 & \frac{26}{3} & 0\end{vmatrix}$ or equivalent with their values

Alternative 4

In 8(b) using area of triangle OBX – area of triangle OCX where X is point (13, 0)

dM1: Uses the correct subtraction $\frac{1}{2}\times 13\times "\frac{26}{3}" - \frac{1}{2}\times 13\times "6"$

Alternative 5

In 8(b) using area = ½ (6 × 4) + ½ (4 × 8/3) drawing a line from C parallel to the x axis and dividing triangle into two right angled triangles

dM1: For correct method area = ½ (“6” × “ 4”) + ½ (“4” × [“26/3”-“6”])

Method 6 Uses calculus

dM1: $\int_0^4 "\frac{26}{3}" - \frac{2x}{3} - \frac{3x}{2}\,dx = \left[\frac{26}{3}x - \frac{x^2}{3} - \frac{3x^2}{4}\right]_0^4$

Question	Scheme	Marks
9(a)	Substitutes $x = 2$ into $y = 20 - 4\times 2 - \frac{18}{2}$ and gets 3	B1
	$\frac{dy}{dx} = -4 + \frac{18}{x^2}$	M1 A1
	Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)	dM1
	States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)	ddM1
	to deduce that $y = -2x + 7$	A1*
		(6)
(b)	Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$ Or put $y = 20 - 4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$	M1 A1
	$(2x - 9)(x - 2) = 0$ so $x =$ **or** $(y - 3)(y + 2) = 0$ so $y =$	dM1
	$\left(\frac{9}{2}, -2\right)$	A1 A1
		(5)
		(11 marks)

Notes:

(a)

B1: Substitutes $x = 2$ into expression for y and gets 3 cao (must be in part (a) and must use curve equation – not line equation). This must be seen to be substituted.

M1: For an attempt to differentiate the negative power with x^{-1} to x^{-2}.

A1: Correct expression for $\frac{dy}{dx} = -4 + \frac{18}{x^2}$

dM1: Dependent on **first** M1 substitutes $x = 2$ into their derivative to obtain a numerical gradient and find negative reciprocal or states that $-2 \times \frac{1}{2} = -1$

Alternative 1

dM1: Dependent on **first** M1. Finds equation of line using changed gradient (not their $\frac{1}{2}$ but $-\frac{1}{2}$ 2 or -2) e.g. $y - "3" = - "2"(x - 2)$ or $y = "-2"\ x$ + c and use of (2, "3") to find $c =$

A1*: cso. This is a given answer $y = -2x + 7$ obtained with no errors seen and equation should be stated.

Alternative 2 – checking given answer

dM1: Uses given equation of line and checks that (2, 3) lies on the line.

A1*: cso. This is a given answer $y = -2x + 7$ so statement that normal and line **have the same gradient** and **pass through the same point** must be stated.

Question 9 notes *continued*
(b) **M1:** Equate the two given expressions, collect terms and simplify to a 3TQ. There may be sign errors when collecting terms but putting for example $20x - 4x^2 - 18 = -2x + 7$ is M0 here. **A1:** Correct 3TQ = 0 (need = 0 for A mark) $2x^2 - 13x + 18 = 0$ **dM1:** Attempt to solve an appropriate quadratic by factorisation, use of formula, or completion of the square (see general instructions). **A1:** $x = \frac{9}{2}$ o.e or $y = -2$ (allow second answers for this mark so ignore $x = 2$ or $y = 3$) **A1:** Correct solutions only so both $x = \frac{9}{2}$, $y = -2$ or $\left(\frac{9}{2}, -2\right)$ If $x = 2$, $y = 3$ is included as an answer and point B is not identified then last mark is A0. Answer only – with no working – send to review. The question stated 'use algebra'.

Question	Scheme		Marks
10(a)	$9^2 = 4^2 + 6^2 - 2\times 4\times 6\cos\alpha \Rightarrow \cos\alpha = ...$	Correct use of cosine rule leading to a value for cos α	M1
	$\cos\alpha = \dfrac{4^2+6^2-9^2}{2\times 4\times 6}\left(= -\dfrac{29}{48} = -0.604..\right)$		
	$\alpha = 2.22$ $*$ cso		A1
			(2)
	Alternative		
	$XY^2 = 4^2 + 6^2 - 2\times 4\times 6\cos 2.22 \Rightarrow XY^2 = ..$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY = 9.00....$		A1
			(2)
(b)	$2\pi - 2.22 (= 4.06366......)$	$2\pi - 2.22$ or $2\pi - 2.2$ or awrt 4.06 (May be implied)	B1
	$\frac{1}{2}\times 4^2 \times "4.06"$	Correct method for major sector area. Allow $\pi - 2.22$ for the major sector angle.	M1
	32.5	Awrt 32.5	A1
			(3)
	Alternative – Circle Minor – sector		
	$\pi \times 4^2$	Correct expression for circle area	B1
	$\pi\times 4^2 - \dfrac{1}{2}\times 4^2 \times 2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$= 32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\dfrac{1}{2}\times 4\times 6\times \sin 2.22 (= 9.56)$	Correct expression for the area of triangle XYZ (allow 2.2 or awrt 2.22)	B1
	So area required = “9.56” + “32.5”	Their Triangle XYZ + part (b) or correct attempt at major sector (**Not** triangle ZXW)	M1
	Area of logo = 42.1 cm^2 or 42.0 cm^2	Awrt 42.1 or 42.0 (or just 42)	A1
			(3)
(d)	Arc length $= 4\times 4.06 (= 16.24)$ **or** $8\pi - 4\times 2.22$	M1: $4\times$*their*$(2\pi - 2.22)$ **or** circumference – minor arc A1: Correct ft expression	M1 A1ft
	Perimeter = $ZY + WY$ + Arc Length	9 + 2 + Any Arc	M1
	Perimeter of logo = 27.2 or 27.3	Awrt 27.2 **or** awrt 27.3	A1
			(4)
			(12 marks)

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WMA12/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = x^4 + x^3 + 2x^2 + ax + b,$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7

(a) Show that $a + b = 3$

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8

(b) Find the value of a and the value of b

(5)

Leave blank

Question 1 continued

Q1

(Total for Question 1 is 7 marks)

Leave blank

2. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$. The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which $S_\infty - S_N < 0.5$

(4)

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

Q2

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3. $$y = \sqrt{(3^x + x)}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(b) Use the trapezium rule with all the values of y from your table to find an approximation for the value of

$$\int_0^1 \sqrt{(3^x + x)}\,\mathrm{d}x$$

You must show clearly how you obtained your answer.

(4)

(c) Explain how the trapezium rule could be used to obtain a more accurate estimate for the value of

$$\int_0^1 \sqrt{(3^x + x)}\,\mathrm{d}x$$

(1)

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Question 3 continued

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Question 3 continued

Q3

(Total for Question 3 is 7 marks)

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4. Given $n \in \mathbb{N}$, prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.

(4)

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Question 4 continued

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Q4

(Total for Question 4 is 4 marks)

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5. An arithmetic series has first term a and common difference d.

(a) Prove that the sum of the first n terms of the series is

$$\tfrac{1}{2}n[2a + (n - 1)d]$$

(4)

A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N.

(b) Find the value of N

(2)

The company then plans to continue to make 600 mobile phones each week.

(c) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1.

(5)

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Question 5 continued

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Question 5 continued

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Question 5 continued

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Q5

(Total for Question 5 is 11 marks)

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6. (i) Find the exact value of x for which

$$\log_2(2x) = \log_2(5x + 4) - 3$$

(4)

(ii) Given that

$$\log_a y + 3\log_a 2 = 5$$

express y in terms of a. Give your answer in its simplest form.

(3)

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Question 6 continued

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Q6

(Total for Question 6 is 7 marks)

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7.

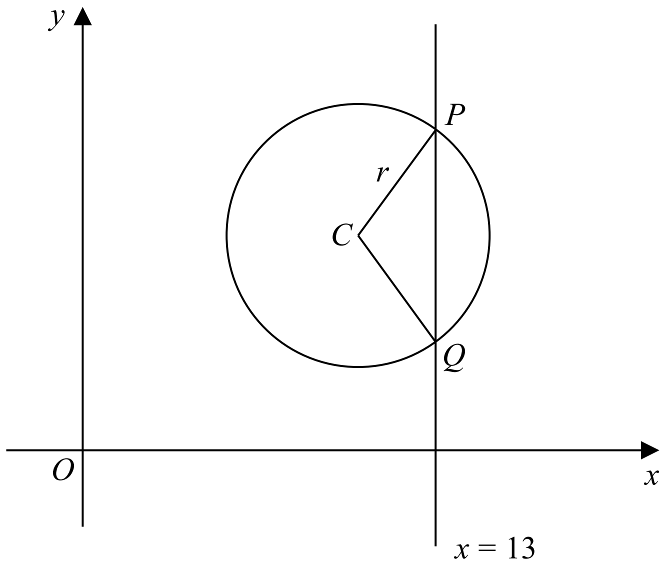

Figure 1

The circle with equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

had centre C and radius r.

(a) Find the coordinates of C.

(2)

(b) Show that $r = 5$

(2)

The line with equation $x = 13$ crosses the circle at the points P and Q as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q.

(3)

A tangent to the circle from O touches the circle at point X.

(d) Find, in surd form, the length OX.

(3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total for Question 7 is 10 marks)

8.

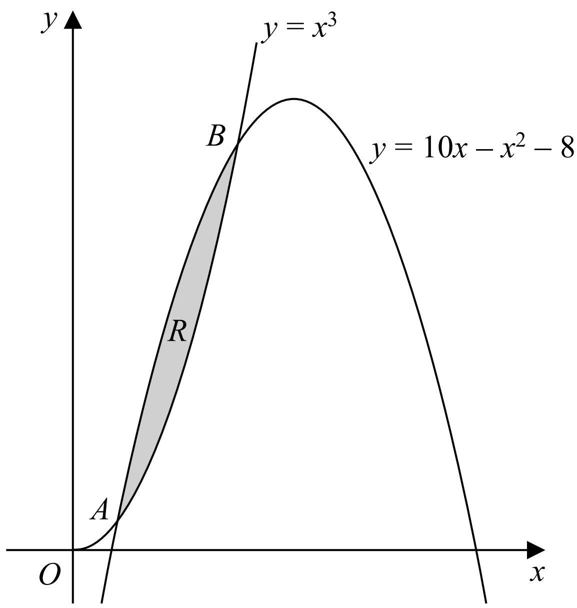

Figure 2

Figure 2 shows a sketch of part of the curves C_1 and C_2 with equations

$$C_1: y = 10x - x^2 - 8 \qquad x > 0$$

$$C_2: y = x^3 \qquad x > 0$$

The curves C_1 and C_2 intersect at the points A and B.

(a) Verify that the point A has coordinates (1, 1)

(1)

(b) Use algebra to find the coordinates of the point B

(6)

The finite region R is bounded by C_1 and C_2

(c) Use calculus to find the exact area of R

(5)

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total for Question 8 is 12 marks)

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9. (i) Solve, for $0 \leqslant \theta < \pi$, the equation

$$\sin 3\theta - \sqrt{3}\cos 3\theta = 0$$

giving your answers in terms of π

(3)

(ii) Given that

$$4\sin^2 x + \cos x = 4 - k, \quad 0 \leqslant k \leqslant 3$$

(a) find $\cos x$ in terms of k

(3)

(b) When $k = 3$, find the values of x in the range $0 \leqslant x < 360°$

(3)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

Pure Mathematics P2 Mark scheme

Question	Scheme	Marks
1(a)	$\mathrm{f}(x) = x^4 + x^3 + 2x^2 + ax + b$	
	Attempting f(1) **or** f(−1)	M1
	f(1) = 1 + 1 + 2 + a + b = 7 **or** 4 + a + b = 7 $\Rightarrow$ a + b = 3 (as required) **AG**	A1* **cso**
		(2)
(b)	Attempting f(−2) **or** f(2)	M1
	f(−2) = 16 – 8 + 8 – 2a + b = –8 {$\Rightarrow$ – 2a + b = –24}	A1
	Solving both equations simultaneously to get as far as a =… **or** b =…	dM1
	Any one of a = 9 **or** b = −6	A1
	Both a = 9 and b = −6	A1
		(5)
		(7marks)

Notes:

(a)

M1: For attempting either $\mathrm{f}(1)$ or $\mathrm{f}(-1)$.

A1: For applying $\mathrm{f}(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a + b = 3$. Note that the answer is given in part (a).

Alternative

M1: For long division by $(x - 1)$ to give a remainder in a and b which is independent of x.

A1: Or $\{\text{Remainder} =\}$ $b + a + 4 = 7$ leading to the correct result of $a + b = 3$ (answer given).

(b)

M1: Attempting either $\mathrm{f}(-2)$ or $\mathrm{f}(2)$.

A1: correct underlined equation in a and b; e.g. $\underline{16 - 8 + 8 - 2a + b = -8}$ or equivalent, e.g. $-2a + b = -24$.

dM1: An attempt to eliminate one variable from 2 linear simultaneous equations in a and b. Note that this mark is dependent upon the award of the first method mark.

A1: Any one of $a = 9$ or $b = -6$.

A1: Both $a = 9$ and $b = -6$ and a correct solution only.

Alternative

M1: For long division by $(x + 2)$ to give a remainder in a and b which is independent of x.

A1: For $\{\text{Remainder} =\}$ $\underline{b - 2(a - 8) = -8}$ $\{\Rightarrow -2a + b = -24\}$.

Then dM1A1A1 are applied in the same way as before.

Question	Scheme		Marks
2(a)	$S_\infty = \dfrac{20}{1-\frac{7}{8}}$; $= 160$	Use of a correct S_∞ formula	M1
		160	A1
			(2)
(b)	$S_{12} = \dfrac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}}$; $= 127.77324...$ $= 127.8$ (1 dp)	M1: Use of a correct S_n formula with n = 12 (condone missing brackets around $\frac{7}{8}$)	M1 A1
		A1: **awrt** 127.8	
			(2)
(c)	$160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$	Applies S_N (**GP only**) with $a = 20$, $r = \frac{7}{8}$ and "uses" 0.5 and their S_∞ at any point in their working.	M1
	$160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$	Attempt to isolate $+160\left(\frac{7}{8}\right)^N$ or $\left(\frac{7}{8}\right)^N$	dM1
	$N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$	Uses the law of logarithms to obtain an equation or an inequality of the form $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ or $N > \log_{0.875}\left(\dfrac{0.5}{\text{their } S_\infty}\right)$	M1
	$N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823...$ **cso** $\Rightarrow N = 44$	$N = 44$ (Allow $N \geq 44$ but no $N > 44$	A1 cso
	An incorrect **<u>inequality</u>** statement at any stage in a candidate's working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. **BUT** it is possible to gain full marks for using =, as long as no incorrect working seen.		
			(4)
	Alternative: Trial & Improvement Method in (c):		
	Attempts $160 - S_N$ or S_N with at least one value for $N > 40$		M1
	Attempts $160 - S_N$ or S_N with $N = 43$ or $N = 44$		dM1
	For evidence of examining $160 - S_N$ or S_N for **both** $N = 43$ **and** $N = 44$ with **both** values correct to 2 DP Eg: $160 - S_{43} =$ awrt 0.51 and $160 - S_{44} =$ awrt 0.45 or $S_{43} =$ awrt 159.49 and $S_{44} =$ awrt 159.55		M1
	$N = 44$		A1 cso
	Answer of N = 44 only with no working scores no marks		
			(4)
			(8 marks)

Question	Scheme	Marks
3(a)	<table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y</td><td>1</td><td>1.251</td><td>**1.494**</td><td>**1.741**</td><td>2</td></tr></table>	B1 B1
		(2)
(b)	$\frac{1}{2}\times 0.25$, $\{(1+2)+2(1.251+1.494+1.741)\}$ o.e.	B1 M1 A1ft
	$= 1.4965$	A1
		(4)
(c)	Gives any valid reason including • Decrease the width of the strips • Use more trapezia • Increase the number of strips Do not accept use more decimal places	B1
		(1)
		(7 marks)

Notes:

(a)

B1: For 1.494

B1: For 1.741 (1.740 is **B0**). Wrong accuracy e.g. 1.49, 1.74 is B1B0

(b)

B1: Need ½ of 0.25 or 0.125 o.e.

M1: Requires first bracket to contain first plus last values **and** second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and **M** mark can be allowed (An extra repeated term forfeits the **M** mark however) x values: **M0** if values used in brackets are x values instead of y values

A1ft: Follows their answers to part (a) and is for {correct expression}

A1: Accept 1.4965, 1.497, or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table).

Separate trapezia may be used: **B1** for 0.125, **M1** for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and **A1**ft if it is all correct) e.g. $0.125(1+1.251) + 0.125(1.251+1.494) + 0.125(1.741+2)$ is **M1 A0** equivalent to missing one term in { } in main scheme.

Question	Scheme	Marks
4	**A solution based around a table of results** <table><tr><td>n</td><td>n^2</td><td>n^2+2</td><td></td></tr><tr><td>**1**</td><td>**1**</td><td>**3**</td><td>**Odd**</td></tr><tr><td>**2**</td><td>**4**</td><td>**6**</td><td>**Even**</td></tr><tr><td>**3**</td><td>**9**</td><td>**11**</td><td>**Odd**</td></tr><tr><td>**4**</td><td>**16**</td><td>**18**</td><td>**Even**</td></tr><tr><td>**5**</td><td>**25**</td><td>**27**</td><td>**Odd**</td></tr><tr><td>**6**</td><td>**36**</td><td>**38**</td><td>**Even**</td></tr></table>	
	When n is odd, n^2 is odd (odd $\times$ odd = odd) so n^2+2 is also odd	M1
	So for all odd numbers n, n^2+2 is also odd and so cannot be divisible by 4 (as all numbers in the 4 times table are even)	A1
	When n is even, n^2 is even **and a multiple** of 4, so n^2+2 cannot be a multiple of 4	M1
	Fully correct and exhaustive proof. Award for both of the cases above plus a final statement "So for all n, n^2+2 cannot be divisible by 4"	A1*
		(4)
	Alternative - (algebraic) proof	
	If n is even, $n=2k$, so $\frac{n^2+2}{4}=\frac{(2k)^2+2}{4}=\frac{4k^2+2}{4}=k^2+\frac{1}{2}$	M1
	If n is odd, $n=2k+1$, so $\frac{n^2+2}{4}=\frac{(2k+1)^2+2}{4}==\frac{4k^2+4k+3}{4}=k^2+k+\frac{3}{4}$	M1
	For a partial explanation stating that • either of $k^2+\frac{1}{2}$ or $k^2+k+\frac{3}{4}$ are not a whole numbers. • with some valid reason stating why this means that n^2 +2 is not a multiple of 4.	A1
	Full proof with no errors or omissions. This must include • The conjecture • Correct notation and algebra for both even and odd numbers • A full explanation stating why, for all n, n^2+2 is not divisible by 4	A1*
		(4)
		(4 marks)

Question	Scheme		Marks
5(a)	$(S=)a+(a+d)+\ldots+[a+(n-1)d]$	B1: requires at least 3 terms, must include first and last terms, an adjacent term and dots!	B1
	$(S=)[a+(n-1)d]+\ldots\ldots+a$	M1: for reversing series (dots needed)	M1
	$2S=[2a+(n-1)d]+\ldots\ldots+[2a+(n-1)d]$	dM1: for adding, must have $2S$ and be a genuine attempt. Either line is sufficient. Dependent on 1st M1.	dM1
	$2S=n[2a+(n-1)d]$ $S=\frac{n}{2}[2a+(n-1)d]$ cso	(NB −Allow first 3 marks for use of l for last term but as given for final mark)	A1
			(4)
(b)	$600=200+(N-1)20 \Rightarrow N=\ldots$	Use of 600 with a **correct** formula in an attempt to find N.	M1
	$N=21$	cso	A1
			(2)
(c)	**Look for an AP first:**		
	$S=\frac{21}{2}(2\times200+20\times20)$ **or** $\frac{21}{2}(200+600)$	M1: Use of correct sum formula with their **integer** $n=N$ or $N-1$ from part (b) where $3<N<52$ and $a=200$ and $d=20$.	M1A1
	$S=\frac{20}{2}(2\times200+19\times20)$ **or** $\frac{20}{2}(200+580)$ (= 8400 **or** 7800)	M1: Use of correct sum formula with their **integer** $n=N$ or $N-1$ from part (b) where $3<N<52$ and $a=200$ and $d=20$.	
	Then for the constant terms:		
	$600\times(52-"N")$ (= 18600)	M1: $600\times k$ where k is an integer and $3<k<52$	M1
		A1: A correct un-simplified follow through expression with their k consistent with n so that $n+k=52$	A1ft
	So total is 27000	**cao**	A1
	There are no marks in (c) for just finding S_{52}		
			(5)
			(11 marks)

Question	Scheme		Marks
6(i)	$\log_2\left(\dfrac{2x}{5x+4}\right)=-3$ **or** $\log_2\left(\dfrac{5x+4}{2x}\right)=3$ **or** $\log_2\left(\dfrac{5x+4}{x}\right)=4$		M1
	$\left(\dfrac{2x}{5x+4}\right)=2^{-3}$ **or** $\left(\dfrac{5x+4}{2x}\right)=2^3$ **or** $\left(\dfrac{5x+4}{x}\right)=2^4$		M1
	$16x=5x+4 \Rightarrow x=$ (depends on Ms and must be this equation or equiv)		dM1
	$x=\dfrac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work		A1 **cso**
	Alternative		
	$\log_2(2x)+3=\log_2(5x+4)$		
	So $\log_2(2x)+\log_2(8)=\log_2(5x+4)$ earns 2nd M1 (3 replaced by $\log_2 8$)		2nd M1
	Then $\log_2(16x)=\log_2(5x+4)$ earns 1st M1 (addition law of logs)		1st M1
	Then final M1 A1 as before		dM1A1
			(4)
(ii)	$\log_a y+\log_a 2^3=5$		M1
	$\log_a 8y=5$	Applies product law of logarithms	dM1
	$y=\dfrac{1}{8}a^5$ **cso**	$y=\dfrac{1}{8}a^5$ **cso**	A1
			(3)
			(7 marks)

Notes:

(i)

M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term .

M1: For RHS of either 2^{-3}, 2^3, 2^4 or $\log_2\left(\dfrac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an error. **Use of 3^2 is M0**

dM1: Obtains **correct** linear equation in x. usually the one in the scheme and attempts $x=$

A1: **cso**. Answer of 4/11 with **no** suspect log work preceding this.

(ii)

M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ or $\log_a 8$

dM1: (Should not be following M0) Uses addition law of logs to give $\log_a 2^3 y=5$ or $\log_a 8y=5$

Question	Scheme	Marks
7(a)	Obtain $(x \pm 10)^2$ **and** $(y \pm 8)^2$	M1
	(10, 8)	A1
		(2)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25\ (= r^2)$ **or** $(r^2 =)\ "100" + "64" - 139$	M1
	$r = 5$*	A1
		(2)
(c)	Substitute $x = 13$ into the equation of circle and solve quadratic to give $y =$	M1
	e.g. $x = 13 \Rightarrow (13 - 10)^2 + (y - 8)^2 = 25 \Rightarrow (y - 8)^2 = 16$ so $y = 4$ **or** 12 **N.B. This can be attempted via a 3, 4, 5 triangle so spotting this and achieving one value for y is M1 A1. Both values scores M1 A1 A1**	A1 A1
		(3)
(d)	$OC = \sqrt{10^2 + 8^2} = \sqrt{164}$	M1
	Length of tangent $= \sqrt{164 - 5^2} = \sqrt{139}$	M1 A1
		(3)
		(10 marks)

Notes:

(a)

M1: Obtains $(x \pm 10)^2$ **and** $(y \pm 8)^2$ May be implied by one correct coordinate

A1: (10, 8) Answer only scores both marks.

Alternative: *Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g,\ \pm f)$

M1: Obtains $(\pm 10, \pm 8)$

A1: Centre is $(-g,\ -f)$, **and so centre is** (10, 8).

(b)

M1: For a correct method leading to $r = \ldots$, or $r^2 =$

Allow $"100" + "64" - 139$ or an attempt at using $(x \pm 10)^2 + (y \pm 8)^2 = r^2$ form to identify $r=$

A1*: $r = 5$ This is a printed answer, so a correct method must be seen.

Alternative:

(b)

M1: Attempts to use $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)"100" + "64" - 139$

A1*: $r = 5$ following a correct method.

(c)

M1: Substitutes $x = 13$ into either form of the circle equation, forms and solves the quadratic equation in y

A1: Either $y = 4$ **or** 12

A1: Both $y = 4$ **and** 12

Question 7 notes *continued*
(d) **M1:** Uses Pythagoras' Theorem to find length OC using their $(10, 8)$ **M1:** Uses Pythagoras' Theorem to find *OX*. Look for $\sqrt{OC^2 - r^2}$ **A1:** $\sqrt{139}$ only

Question	Scheme	Marks
8(a)	Substitutes $x=1$ in C_1: $y = 10x - x^2 - 8 = 10 - 1 - 8 = 1$ and in C_2: $y = x^3 = 1^3 = 1 \Rightarrow (1, 1)$ lies on both curves.	B1
		(1)
(b)	$10x - x^2 - 8 = x^3$ $x^3 + x^2 - 10x + 8 = 0$	B1
	$(x-1)(x^2+2x-8) = 0$	M1 A1
	$(x-1)(x+4)(x-2) = 0 \qquad x = 2$	M1 A1
	$(2, 8)$	A1
		(6)
(c)	$\int \left\{ \left(10x - x^2 - 8\right) - x^3 \right\} \mathrm{d}x$	M1
	$= 5x^2 - \frac{x^3}{3} - 8x - \frac{x^4}{4}$	M1 A1
	Using limits 2 and 1: $\left(20 - \frac{8}{3} - 16 - 4\right) - \left(5 - \frac{1}{3} - 8 - \frac{1}{4}\right)$	M1
	$= \frac{11}{12}$	A1
		(5)
		(12 marks)

Notes:

(a)

B1: Substitutes x =nto both $y = 10x - x^2 - 8$ and $y = x^3$ **AND** achieves y =1 in both.

(b)

B1: Sets equations equal to each other and proceeds to $x^3 + x^2 - 10x + 8 = 0$

M1: Divides by $(x - 1)$ to form a quadratic factor. Allow any suitable algebraic method including division or inspection.

A1: Correct quadratic factor $(x^2 + 2x - 8)$

M1: For factorising of their quadratic factor.

A1: Achieves x= 2

A1: Coordinates of $B = (2, 8)$

(c)

M1: For knowing that the area of R = $\int \left\{ \left(10x - x^2 - 8\right) - x^3 \right\} \mathrm{d}x$

This may also be scored for finding separate areas and subtracting.

M1: For raising the power of x seen in at least three terms.

A1: Correct integration. It may be left un-simplified. That is allow $\frac{10x^2}{2}$ for $5x^2$

Question 8 notes *continued*	
M1:	For using the limits "2" and 1 in their integrated expression. If separate areas have been attempted, "2" and 1 must be used in both integrated expressions.
A1:	For $\frac{11}{12}$ or exact equivalent.

Question	Scheme		Marks
9(i)	**Way 1** Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so$\Rightarrow (3\theta) = \frac{\pi}{3}$	**Way 2** Or Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = \frac{\pi}{3}$	M1
	Adds π **or** 2π to previous value of angle(to give $\frac{4\pi}{3}$ **or** $\frac{7\pi}{3}$)		M1
	So $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ (all three, no extra in range)		A1
			(3)
(ii)(a)	$4(1 - \cos^2 x) + \cos x = 4 - k$	Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$		dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ **or** $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ **or** other correct equivalent		A1
			(3)
(b)	$\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made)		M1
	Obtains two solutions from 0 , 139 , 221 (0 **or** 2.42 **or** 3.86 in radians)		dM1
	$x = 0$ and 139 and 221 (allow awrt 139 and 221) must be in degrees		A1
			(3)
			(9 marks)

Notes:

(i)

M1: Obtains $\frac{\pi}{3}$. Allow $x = \frac{\pi}{3}$ or even $\theta = \frac{\pi}{3}$. Need not see working here. May be implied by $\theta = \frac{\pi}{9}$ in final answer (allow $(3\theta) = 1.05$ or $\theta = 0.349$ as decimals or $(3\theta) = 60$ or $\theta = 20$ as degrees for this mark). Do not allow $\tan 3\theta = -\sqrt{3}$ nor $\tan 3\theta = \pm\frac{1}{\sqrt{3}}$

M1: Adding π or 2π to a previous value however obtained. It is not dependent on the previous mark. (May be implied by final answer of $\theta = \frac{4\pi}{9}$ **or** $\frac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees (240 or 420).

Question 9 notes *continued*
A1: Need all three correct answers in terms of π and **no extras in range**. **NB:** $\theta = 20°$,$80°$, $140°$ **earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0**
(ii)(a) **M1:** Applies $\sin^2 x = 1 - \cos^2 x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos^2 x$). This must be awarded in (ii) (a) for an expression with *k* not after *k* = 3 is substituted. **dM1:** Uses formula or completion of square to obtain cos *x* = expression in *k* (Factorisation attempt is M0) **A1:** cao - award for their final simplified expression
(ii)(b) **M1:** **Either** attempts to substitute *k* = 3 into their answer to obtain two values for cos *x* **Or** restarts with *k* = 3 to find two values for cos *x* (They cannot earn marks in ii(a) for this). **In both cases** they need to have applied $\sin^2 x = 1 - \cos^2 x$ (brackets may be missing) **and** correct method for solving their quadratic (usual rules – see notes) The values for cos *x* may be >1 or < -1. **dM1:** Obtains **two correct** values for *x* **A1:** Obtains **all three correct values** in degrees (allow awrt 139 and 221) including 0. Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Mathematics

International Advanced Level
Pure Mathematics P3

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WMA13/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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S 5 9 7 5 7 A 0 1 3 5

Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. Express

$$\frac{6x + 4}{9x^2 - 4} - \frac{2}{3x + 1}$$

as a single fraction in its simplest form.

(4)

Question 1 continued

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Q1

(Total for Question 1 is 4 marks)

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2. $$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)} \quad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)} \quad n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3

(3)

The root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(2)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total for Question 2 is 8 marks)

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3.

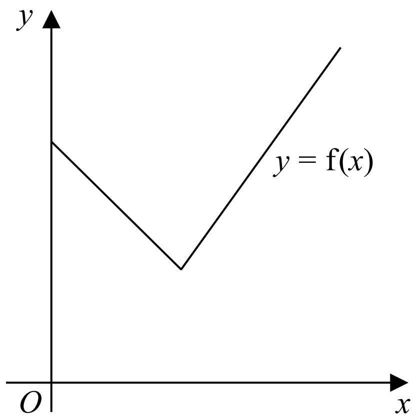

Figure 1

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2|3 - x| + 5 \qquad x \geqslant 0$$

(a) Solve the equation

$$\mathrm{f}(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has two distinct roots,

(b) state the set of possible values for k.

(2)

Leave blank

Question 3 continued

Q3

(Total for Question 3 is 5 marks)

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4. (i) Find

$$\int_{5}^{13} \frac{1}{(2x-1)}\,\mathrm{d}x$$

writing your answer in its simplest form.

(4)

(ii) Use integration to find the exact value of

$$\int_{0}^{\frac{\pi}{2}} \sin 2x + \sec\frac{1}{3}x\tan\frac{1}{3}x\,\mathrm{d}x$$

(3)

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Question 4 continued

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Question 4 continued

Q4

(Total for Question 4 is 7 marks)

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5. Given that

$$y = \frac{5x^2 - 10x + 9}{(x-1)^2} \qquad x \neq 1$$

show that $\dfrac{dy}{dx} = \dfrac{k}{(x-1)^3}$, where k is a constant to be found.

(6)

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Question 5 continued

Q5

(Total for Question 5 is 6 marks)

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6. The functions f and g are defined by

$$\mathrm{f}: x \mapsto \mathrm{e}^x + 2 \qquad x \in \mathbb{R}$$

$$\mathrm{g}: x \mapsto \ln x \qquad x > 0$$

(a) State the range of f.

(1)

(b) Find fg(x), giving your answer in its simplest form.

(2)

(c) Find the exact value of x for which $\mathrm{f}(2x + 3) = 6$

(4)

(d) Find f^{-1} stating its domain.

(3)

(e) On the same axes sketch the curves with equation $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

Question 6 continued

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Question 6 continued

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Question 6 continued

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Q6

(Total for Question 6 is 14 marks)

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7. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \dfrac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total for Question 7 is 7 marks)

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8. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

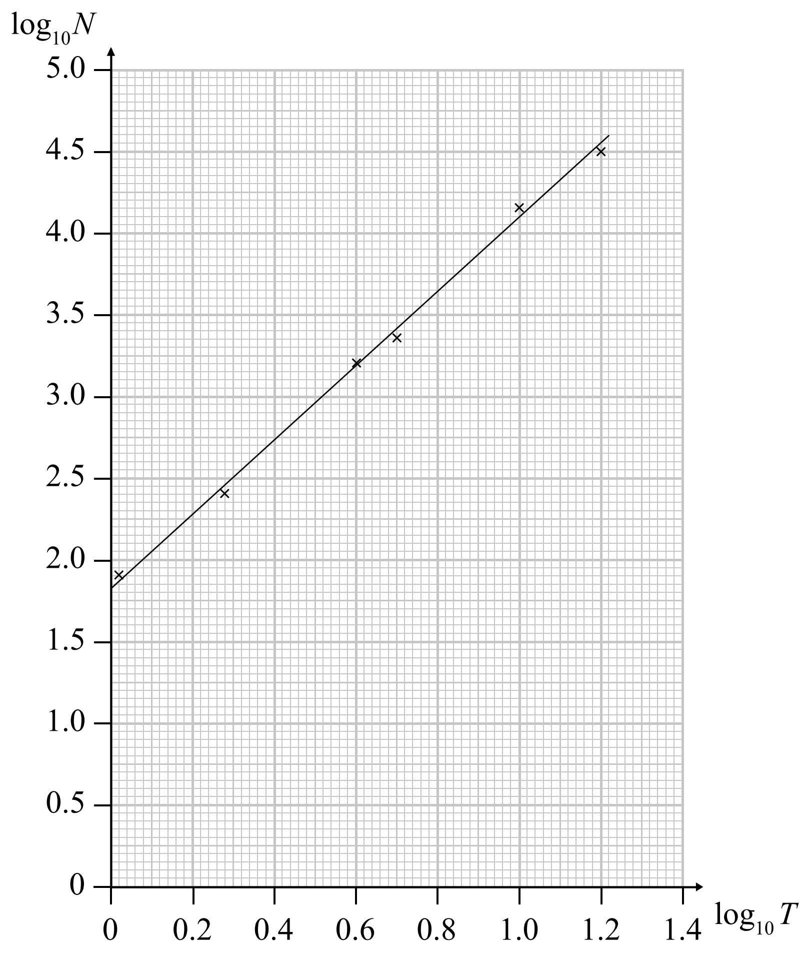

Figure 2

Figure 2 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) With reference to the model, interpret the value of the constant a.

(1)

Leave blank

Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total for Question 8 is 7 marks)

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9. (a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A} \qquad A \neq \frac{(2n+1)\pi}{4} \qquad n \in \mathbb{Z}$$

(5)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give *y*our answers to 3 decimal places.

(4)

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Leave blank

Question 9 continued

Q9

(Total for Question 9 is 9 marks)

Leave blank

10. The amount of an antibiotic in the bloodstream, from a given dose, is modelled by the formula

$$x = De^{-0.2t}$$

where x is the amount of the antibiotic in the bloodstream in milligrams, D is the dose given in milligrams and t is the time in hours after the antibiotic has been given.

A first dose of 15 mg of the antibiotic is given.

(a) Use the model to find the amount of the antibiotic in the bloodstream 4 hours after the dose is given. Give your answer in mg to 3 decimal places.

(2)

A second dose of 15 mg is given 5 hours after the first dose has been given. Using the same model for the second dose,

(b) show that the total amount of the antibiotic in the bloodstream 2 hours after the second dose is given is 13.754 mg to 3 decimal places.

(2)

No more doses of the antibiotic are given. At time T hours after the second dose is given, the total amount of the antibiotic in the bloodstream is 7.5 mg.

(c) Show that $T = a \ln\left(b + \frac{b}{e}\right)$, where a and b are integers to be determined.

(4)

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Question 10 continued

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Question 10 continued

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Question 10 continued

Q10

(Total for Question 10 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Pure Mathematics P3 Mark scheme

Question	Scheme	Marks
1	$9x^2 - 4 = (3x - 2)(3x + 2)$ at any stage	B1
	Eliminating the common factor of $(3x + 2)$ at any stage $\dfrac{2\cancel{(3x+2)}}{(3x-2)\cancel{(3x+2)}} = \dfrac{2}{3x-2}$	M1
	Use of a common denominator $\dfrac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)} - \dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ **or** $\dfrac{2(3x+1)}{(3x-2)(3x+1)} - \dfrac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\dfrac{6}{(3x-2)(3x+1)}$ **or** $\dfrac{6}{9x^2-3x-2}$	A1
		(4)
		(4 marks)

Notes:

B1: For factorising $9x^2 - 4 = (3x-2)(3x+2)$ using difference of two squares. It can be awarded at any stage of the answer but it must be scored on E pen as the first mark.

B1: For eliminating/cancelling out a factor of $(3x+2)$ at any stage of the answer.

M1: For combining two fractions to form a single fraction with a common denominator. Allow slips on the numerator but at least one must have been adapted. Condone invisible brackets. Accept two separate fractions with the same denominator as shown in the mark scheme. Amongst possible (incorrect) options scoring method marks are

$\dfrac{2(3x+2)}{(9x^2-4)(3x+1)} - \dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$. Only one numerator adapted, separate fractions

$\dfrac{2\times 3x+1-2\times 3x-2}{(3x-2)(3x+1)}$ Invisible brackets, single fraction.

A1: $\dfrac{6}{(3x-2)(3x+1)}$

This is not a given answer so you can allow recovery from 'invisible' brackets.

Alternative

$$\frac{2(3x+2)}{(9x^2-4)} - \frac{2}{(3x+1)} = \frac{2(3x+2)(3x+1) - 2(9x^2-4)}{(9x^2-4)(3x+1)} = \frac{18x+12}{(9x^2-4)(3x+1)}$$ has scored 0,0,1,0 so far

$$= \frac{6\cancel{(3x+2)}}{\cancel{(3x+2)}(3x-2)(3x+1)}$$ is now 1,1,1,0

$$= \frac{6}{(3x-2)(3x+1)}$$ and now 1,1,1,1

Question	Scheme	Marks
2(a)	$x^3 + 3x^2 + 4x - 12 = 0 \Rightarrow x^3 + 3x^2 = 12 - 4x$	
	$\Rightarrow x^2(x + 3) = 12 - 4x$	M1
	$\Rightarrow x^2 = \dfrac{12-4x}{(x+3)}$ $\Rightarrow x = \sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1 A1*
		(3)
(b)	$x_1 = \sqrt{\left(\dfrac{4(3-1)}{(3+1)}\right)} = 1.41$	M1 A1
	awrt $x_2 = 1.20 \quad x_3 = 1.31$	A1
		(3)
(c)	Attempts $\mathrm{f}(1.2725) = (+)0.00827... \quad \mathrm{f}(1.2715) = -0.00821....$	M1
	Values correct with reason (change of sign with f(x) continuous) and conclusion ($\Rightarrow \alpha = 1.272$)	A1
		(2)
		(8 marks)

Notes:

(a)

M1: Moves from f(x) = 0, which may be implied by subsequent working, to $x^2(x \pm 3) = \pm 12 \pm 4x$ by separating terms and factorising in either order. No need to factorise rhs for this mark.

dM1: Divides by '(x+3)' term to make x^2 the subject, then takes square root. No need for rhs to be factorised at this stage.

A1*: CSO. This is a given solution. Do not allow sloppy algebra or notation with root on just numerator for instance. The 12−4x needs to have been factorised.

(b)

M1: An attempt to substitute $x_0 = 1$ into the iterative formula to calculate x_1. This can be awarded fo the sight of $\sqrt{\dfrac{4(3-1)}{(3+1)}}, \sqrt{\dfrac{8}{4}}, \sqrt{2}$ and even 1.4

A1: $x_1 = 1.41$. The subscript is not important. Mark as the first value found, $\sqrt{2}$ is A0

A1: $x_2 = \text{awrt } 1.20 \quad x_3 = \text{awrt } 1.31$. Mark as the second and third values found. Condone 1.2 for x_2

(c)

M1: Calculates f(1.2715) and f(1.2725), or **the** tighter interval with at least 1 correct to 1 sig fig rounded or truncated. Accept f(1.2715) = -0.008 1sf rounded or truncated. Also accept f(1.2715) = -0.01 2dp. Accept f(1.2725) = (+) 0.008 1sf rounded or truncated. Also accept f(1.2725) = (+)0.01 2dp

A1: Both values correct (see above),
A valid reason; Accept change of sign, or >0 <0, or f(1.2715) ×f(1.2725)<0
And a (minimal) conclusion; Accept hence root or α=1.272 or QED or □

Question	Scheme	Marks
3(a)	Uses $-2(3-x)+5=\frac{1}{2}x+30$	M1
	Attempts to solve by multiplying out bracket, collect terms etc. $\frac{3}{2}x=31$	M1
	$x=\frac{62}{3}$ only	A1
		(3)
(b)	Makes the connection that there must be two intersections. Implied by either end point $k>5$ **or** $k\leqslant 11$	M1
	$5<k\leqslant 11$	A1
		(2)
		(5 marks)

Notes:

(a)

M1: Deduces that the solution to $f(x)=\frac{1}{2}x+30$ can be found by solving $-2(3-x)+5=\frac{1}{2}x+30$

M1: Correct method used to solve their equation. Multiplies out bracket/ collects like terms.

A1: $x=\frac{62}{3}$ only. Do not allow 20.6

(b)

M1: Deduces that two distinct roots occurs when $y=k$ intersects $y=f(x)$ in two places. This may be implied by the sight of either end point. Score for sight of either $k>5$ or $k\leqslant 11$

A1: Correct solution only $\{k:k\in\mathbb{R},5<k\leqslant 11\}$

Question	Scheme		Marks
4(i)	$\int \frac{1}{(2x-1)}\,dx = \frac{1}{2}\ln(2x-1)$		M1 A1
	$\int_5^{13} \frac{1}{(2x-1)}\,dx = \frac{1}{2}\ln 25 - \frac{1}{2}\ln 9 = \frac{1}{2}\ln\left(\frac{25}{9}\right)$		dM1
	$= \ln\left(\frac{5}{3}\right)$		A1
			(4)
(ii)	Integrates to give $\alpha\cos 2x + \beta\sec\frac{1}{3}x\{+c\}$ where $\alpha \neq 0, \beta \neq 0$ $\left[-\frac{1}{2}\cos 2x + 3\sec\frac{1}{3}x\{+c\}\right]$		M1
	$\left(-\frac{1}{2}\cos\left(2\times\frac{\pi}{2}\right)+3\sec\left(\frac{1}{3}\times\frac{\pi}{2}\right)\right)-\left(-\frac{1}{2}\cos(0)+3\sec(0)\right)$	Substitutes limits of 0 and $\frac{\pi}{2}$ and subtracts the correct way around	dM1
	$= 2\sqrt{3} - 2$		A1
			(3)
			(7 marks)

Notes:

(i)

M1: For $\int \frac{1}{(2x-1)}\,dx = k\ln(2x-1)$ where k is a constant.

A1: Correct integration $\int \frac{1}{(2x-1)}\,dx = \frac{1}{2}\ln(2x-1)$

dM1: Scored for substituting in the limits, subtracting and using correctly at least one log law.

You may see the subtraction law $k\ln 25 - k\ln 9 = k\ln\left(\frac{25}{9}\right)$ or the index law

$\frac{1}{2}\ln 25 - \frac{1}{2}\ln 9 = \ln 5 - \ln 3$

A1: cao $\ln\left(\frac{5}{3}\right)$

(ii)

M1: Integrates to a form $\alpha\cos 2x + \beta\sec\frac{1}{3}x\{+c\}$ where $\alpha \neq 0, \beta \neq 0$

dM1: Dependent upon the previous M1. It is scored for substituting limits of 0 and $\frac{\pi}{2}$ and subtracting the correct way around.

A1: cao $2\sqrt{3} - 2$

Question	Scheme	Marks
5	$y=\dfrac{5x^2-10x+9}{(x-1)^2}$	
	Differentiates numerator to $10x-10$ and denominator to $2(x-1)$ o.e.	B1
	Uses the quotient rule $$\frac{dy}{dx}=\frac{(x-1)^2(10x-10)-(5x^2-10x+9)2(x-1)}{(x-1)^4}$$	M1 A1
	Takes out a common factor from the numerator and cancels $$\frac{dy}{dx}=\frac{\cancel{(x-1)}\{(x-1)(10x-10)-(5x^2-10x+9)2\}}{(x-1)^{\cancel{4}3}}$$	M1
	Simplifies the numerator by multiplying and collecting terms $$\frac{dy}{dx}=\frac{\{10x^2-20x+10-10x^2+20x-18\}}{(x-1)^3}$$	M1
	$$\frac{dy}{dx}=\frac{-8}{(x-1)^3}$$	A1
		(6)
		(6 marks)

Notes:

B1: See scheme.

M1: Uses the quotient rule to reach a form $\dfrac{dy}{dx}=\dfrac{(x-1)^2(Ax+B)-(5x^2-10x+9)(Cx+D)}{(x-1)^4}$ o.e.

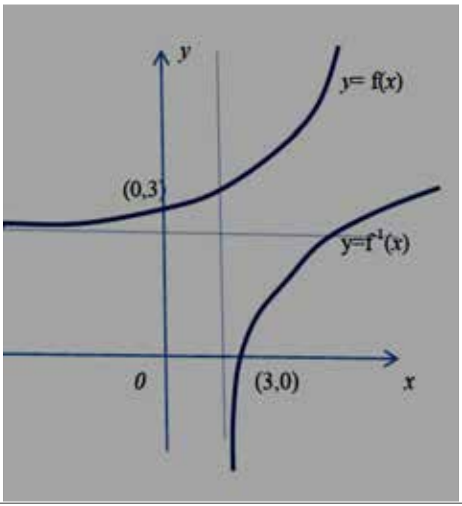
Alternatively uses the product rule to reach a for
$\dfrac{dy}{dx}=(x-1)^{-2}(Ax+B)+(5x^2-10x+9)C(x-1)^{-3}$

A1: Fully correct $\dfrac{dy}{dx}$ If the product rule is used
$\dfrac{dy}{dx}=(x-1)^{-2}(10x-10)-(5x^2-10x+9)2(x-1)^{-3}$

M1: This is for using a correct method to reach a form $\dfrac{dy}{dx}=\dfrac{g(x)}{(x-1)^3}$. See scheme when using the quotient rule. If the product rule is used it is for combining the terms using a common denominator.

M1: Scored for simplifying the numerator (By multiplying out and collecting terms).

A1: $\dfrac{dy}{dx}=\dfrac{-8}{(x-1)^3}$

Question	Scheme	Marks
6(a)	$\mathrm{f}(x) > 2$	B1
		(1)
(b)	$\mathrm{fg}(x) = e^{\ln x} + 2\,, = x + 2$	M1 A1
		(2)
(c)	$e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$	M1 A1
	$\Rightarrow 2x + 3 = \ln 4$	
	$\Rightarrow x = \dfrac{\ln 4 - 3}{2}$ or $\ln 2 - \dfrac{3}{2}$	M1 A1
		(4)
(d)	Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y-2) = x$	M1
	$\mathrm{f}^{-1}(x) = \ln(x-2),\ x > 2$	A1 B1ft
		(3)
(e)	 Shape for $\mathrm{f}(x)$	B1
	(0, 3)	B1
	Shape for $\mathrm{f}^{-1}(x)$	B1
	(3, 0)	B1
		(4)
		(14 marks)

Notes:

(a)

B1: Range of f(x)>2. Accept y>2, (2,∞), f>2, as well as 'range is the set of numbers bigger than 2' but **don't accept** x >2

(b)

M1: For applying the correct order of operations. Look for $e^{\ln x} + 2$. Note that $\ln e^x + 2$ is M0

A1: Simplifies $e^{\ln x} + 2$ to $x + 2$. Just the answer is acceptable for both marks.

(c)

M1: Starts with $e^{2x+3} + 2 = 6$ and proceeds to $e^{2x+3} = ...$

A1: $e^{2x+3} = 4$

M1: Takes ln's both sides, $2x + 3 = \ln ..$ and proceeds to x=….

Question 6 notes *continued*	
A1:	$x = \dfrac{\ln 4 - 3}{2}$ oe. eg $\ln 2 - \dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer.
(d)	
M1:	Starts with $y = e^x + 2$ or $x = e^y + 2$ and attempts to change the subject. All ln work must be correct. The 2 must be dealt with first. Eg. $y = e^x + 2 \Rightarrow \ln y = x + \ln 2 \Rightarrow x = \ln y - \ln 2$ is M0.
A1:	$\mathrm{f}^{-1}(x) = \ln(x-2)$ or $y = \ln(x-2)$ or $y = \ln\lvert x-2\rvert$ There must be some form of bracket.
B1ft:	Either $x > 2$, or follow through on their answer to part (a), provided that it wasn't $y \in \Re$ Do not accept $y>2$ or $\mathrm{f}^{-1}(x)>2$.
(e)	
B1:	Shape for $y = e^x$. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the x axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.
B1:	(0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve.
B1:	Shape for $y = \ln x$. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the y axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects $y = e^x$.
B1:	(3, 0) lies on the curve. Accept 3 written on the x axis as long as the point lies on the curve.

Question	Scheme	Marks
7(a)	$p = 4\pi^2$ **or** $(2\pi)^2$	B1
		(1)
(b)	$x = (4y - \sin 2y)^2 \Rightarrow \dfrac{dx}{dy} = 2(4y - \sin 2y)\,(4 - 2\cos 2y)$	M1 A1
	Sub $y = \dfrac{\pi}{2} \Rightarrow \dfrac{dx}{dy} = 24\pi$ (= 75.4) OR $\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{24\pi}$(= 0.013)	M1
	Equation of tangent $y - \dfrac{\pi}{2} = \dfrac{1}{24\pi}\ \ x - 4\pi^2$	M1
	Using $y - \dfrac{\pi}{2} = \dfrac{1}{24\pi}\ \ x - 4\pi^2$ with $x = 0 \Rightarrow y = \dfrac{\pi}{3}$ **cso**	M1 A1
		(6)
	Alternative I for first two marks	
	$x = (4y - \sin 2y)^2 \Rightarrow x^{0.5} = 4y - \sin 2y$ $\Rightarrow 0.5x^{-0.5}\dfrac{dx}{dy} = 4 - 2\cos 2y$	M1A1
	Alternative II for first two marks	
	$x = \left(16y^2 - 8y\sin 2y + \sin^2 2y\right)$ $\Rightarrow 1 = 32y\dfrac{dy}{dx} - 8\sin 2y\dfrac{dy}{dx} - 16y\cos 2y\dfrac{dy}{dx} + 4\sin 2y\cos 2y\dfrac{dy}{dx}$ **Or** $1dx = 32y\,dy - 8\sin 2y\,dy - 16y\cos 2y\,dy + 4\sin 2y\cos 2y\,dy$	M1A1
		(7 marks)

Notes:

(a)

B1: $p = 4\pi^2$ or exact equivalent $2\pi\ ^2$. Also allow $x = 4\pi^2$

(b)

M1: Uses the chain rule of differentiation to get a form $A(4y - \sin 2y)(B \pm C\cos 2y)$, $A, B, C \neq 0$ on the right hand side.

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x = \left(16y^2 - 8y\sin 2y + \sin^2 2y\right) \Rightarrow \dfrac{dx}{dy} = Py \pm Q\sin 2y \pm Ry\cos 2y \pm S\sin 2y\cos 2y \quad P, Q, R, S \neq 0$

A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5}... = 4 - Q\cos 2y$

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

Question 7 notes *continued*	
A1:	$\frac{\mathrm{d}x}{\mathrm{d}y}=2(4y-\sin 2y)(4-2\cos 2y)$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2(4y-\sin 2y)(4-2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs. In the alternative $\frac{\mathrm{d}x}{\mathrm{d}y}=32y-8\sin 2y-16y\cos 2y+4\sin 2y\cos 2y$
M1:	**S**ub $y=\frac{\pi}{2}$ into their $\frac{\mathrm{d}x}{\mathrm{d}y}$ or inverted $\frac{\mathrm{d}x}{\mathrm{d}y}$. Evidence could be minimal, eg $y=\frac{\pi}{2}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=...$ It is not dependent upon the previous M1 but it must be a changed $x=(4y-\sin 2y)^2$
M1:	Score for a correct method for finding the equation of the tangent at $\left('4\pi^2',\frac{\pi}{2}\right)$. Allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical } {}^{\mathrm{d}x}/_{\mathrm{d}y}}\,x-\text{their }4\pi^2$ Allow for $\left(y-\frac{\pi}{2}\right)\times\text{their numerical } {}^{\mathrm{d}x}/_{\mathrm{d}y}=x-\text{their }4\pi^2$ Even allow for $y-\frac{\pi}{2}=\frac{1}{\text{their numerical } {}^{\mathrm{d}x}/_{\mathrm{d}y}}\,x-p$ It is possible to score this by stating the equation $y=\frac{1}{24\pi}x+c$ as long as $\left('4\pi^2',\frac{\pi}{2}\right)$ is used in a subsequent line.
M1:	Score for writing their equation in the form $y=mx+c$ and stating the value of 'c' **or** setting $x=0$ in their $y-\frac{\pi}{2}=\frac{1}{24\pi}\,x-4\pi^2$ and solving for y. Alternatively using the gradient of the line segment AP = gradient of tangent. Look for $\frac{\frac{\pi}{2}-y}{4\pi^2}=\frac{1}{24\pi}\Rightarrow y=..$ Such a method scores the previous M mark as well. At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.
A1:	**cso** $y=\frac{\pi}{3}$. You do not have to see $\left(0,\frac{\pi}{3}\right)$

Question	Scheme	Marks
8(a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1
		(2)
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** b = gradient	M1
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** b = gradient	M1
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1
	Number of microbes ≈ 800	A1
		(4)
(c)	States that 'a' is the number of microbes 1 day after the start of the experiment.	B1
		(1)
		(7 marks)

Notes:

(a)

M1: Takes $\log_{10}'s$ of both sides and attempts to use the addition law. Condone $\log = \log_{10}$ for this mark.

A1: Proceeds correctly to $\log_{10} N = \log_{10} a + b\log_{10} T$ **and** states $m = b$ and $c = \log_{10} a$

(b) **Way One: Main scheme**

M1: For attempting to use the graph to find either a or b using $a = 10^{\text{intercept}}$ **or** b = gradient. This may be implied by $a = 10^{1.75 \, to \, 1.85}$ or b = 2.27 to 2.33

M1: For attempting to use the graph to find BOTH a and b (See previous M1)

M1: Uses $T = 3$ in $N = aT^b$ with their a and b

A1: Number of microbes ≈ 800

Way Two: Alternative using line of best fit techniques.

M1: For $\log_{10} 3 \approx 0.48$ and using the graph to find $\log_{10} N$

M1: For using the graph to find $\log_{10} N$ (FYI $\log_{10} N \approx 2.9$)

M1: For $\log_{10} N = k \Rightarrow N = 10^k$

A1: Number of microbes ≈ 800

(c)

B1: See scheme.

Question	Scheme	Marks
9(a)	$\sec 2A + \tan 2A = \dfrac{1}{\cos 2A} + \dfrac{\sin 2A}{\cos 2A}$	B1
	$= \dfrac{1+\sin 2A}{\cos 2A}$	M1
	$= \dfrac{1+2\sin A\cos A}{\cos^2 A - \sin^2 A}$	M1
	$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$	
	$= \dfrac{(\cos A + \sin A)(\cos A + \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$	M1
	$= \dfrac{\cos A + \sin A}{\cos A - \sin A}$	A1*
		(5)
(b)	$\sec 2\theta + \tan 2\theta = \dfrac{1}{2} \Rightarrow \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \dfrac{1}{2}$	
	$\Rightarrow 2\cos\theta + 2\sin\theta = \cos\theta - \sin\theta$	
	$\Rightarrow \tan\theta = -\dfrac{1}{3}$	M1 A1
	$\Rightarrow \theta =$ awrt 2.820, 5.961	dM1 A1
		(4)
		(9 marks)

Notes:

(a)

B1: A correct identity for $\sec 2A = \dfrac{1}{\cos 2A}$ **or** $\tan 2A = \dfrac{\sin 2A}{\cos 2A}$.

It need not be in the proof and it could be implied by the sight of $\sec 2A = \dfrac{1}{\cos^2 A - \sin^2 A}$

M1: For setting their expression as a single fraction. The denominator must be correct for their fractions and at least two terms on the numerator.

This is usually scored for $\dfrac{1+\cos 2A \tan 2A}{\cos 2A}$ or $\dfrac{1+\sin 2A}{\cos 2A}$

M1: For getting an expression in just sin A and cos A by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\cos 2A = \cos^2 A - \sin^2 A$, $2\cos^2 A - 1$ or $1 - 2\sin^2 A$.

Alternatively for getting an expression in just sin A and cos A by using the double angle identities $\sin 2A = 2\sin A\cos A$ and $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ with $\tan A = \dfrac{\sin A}{\cos A}$.

For example $= \dfrac{1}{\cos^2 A - \sin^2 A} + \dfrac{2\sin A/\cos A}{1 - \sin^2 A/\cos^2 A}$ is B1M0M1 so far

Question 9 notes *continued*	
M1:	In the main scheme it is for replacing 1 by $\cos^2 A + \sin^2 A$ **and** factorising both numerator and denominator.
A1*:	Cancelling to produce given answer with no errors. Allow a consistent use of another variable such as θ, but mixing up variables will lose the A1*.
(b)	
M1:	For using part (a), cross multiplying, dividing by $\cos\theta$ to reach $\tan\theta = k$ Condone $\tan 2\theta = k$ for this mark only.
A1:	$\tan\theta = -\frac{1}{3}$
dM1:	Scored for $\tan\theta = k$ leading to at least one value (with 1 dp accuracy) for θ between 0 and 2π. You may have to use a calculator to check. Allow answers in degrees for this mark.
A1:	$\theta =$ awrt 2.820, 5.961 with no extra solutions within the range. Condone 2.82 for 2.820. You may condone different/ mixed variables in part (b)

Question	Scheme	Marks
10(a)	Subs $D=15$ and $t=4$ $\quad x = 15e^{-0.2\times4} = 6.740$ (mg)	M1 A1
		(2)
(b)	$15e^{-0.2\times7} + 15e^{-0.2\times2} = 13.754$ (mg)	M1 A1*
		(2)
(c)	$15e^{-0.2\times T} + 15e^{-0.2\times(T+5)} = 7.5$	M1
	$15e^{-0.2\times T} + 15e^{-0.2\times T}e^{-1} = 7.5$	
	$15e^{-0.2\times T}(1+e^{-1}) = 7.5 \Rightarrow e^{-0.2\times T} = \dfrac{7.5}{15\left(1+e^{-1}\right)}$	dM1
	$T = -5\ln\left(\dfrac{7.5}{15\left(1+e^{-1}\right)}\right) = 5\ln\left(2+\dfrac{2}{e}\right)$	A1 A1
		(4)
		(8 marks)

Notes:

(a)

M1: Attempts to substitute both $D=15$ and $t=4$ in $x = De^{-0.2t}$. It can be implied by sight of $15e^{-0.8}$, $15e^{-0.2\times4}$ or awrt 6.7. Condone slips on the power. Eg you may see -0.02

A1: Cao. 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1: Attempt to find the sum of two expressions with $D=15$ in both terms with t values of 2 and 7. Evidence would be $15e^{-0.2\times7} + 15e^{-0.2\times2}$ or similar expressions such as $\left(15e^{-1}+15\right)e^{-0.2\times2}$. Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**. Alternatively finds the amount after 5 hours, $15e^{-1}$ = awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $e^{-0.4}$ to get awrt **13.75**. Sight of 5.52+15=20.52 $\rightarrow$ 13.75 is fine.

A1*: Cso so both the expression $15e^{-0.2\times7} + 15e^{-0.2\times2}$ and $13.754(mg)$ are required

Alternatively both the expression $\left(15e^{-0.2\times5}+15\right)\times e^{-0.2\times2}$ and 13.754 (*mg*) are required.

Sight of just the numbers is not enough for the A1*

(c)

M1: Attempts to write down a correct equation involving T or t. Accept with or without correct bracketing Eg. accept $15e^{-0.2\times T} + 15e^{-0.2\times(T\pm5)} = 7.5$ or similar equations $\left(15e^{-1}+15\right)e^{-0.2\times T} = 7.5$

dM1: Attempts to solve their equation, dependent upon the previous mark, by proceeding to $e^{-0.2\times T} = ..$ An attempt should involve an attempt at the index law $x^{m+n} = x^m \times x^n$ and taking out a factor of $e^{-0.2\times T}$ Also score for candidates who make $e^{+0.2\times T}$ the subject using the same criteria.

Question 10 notes *continued*
A1: Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15\left(1+e^{-1}\right)}\right)$ **A1:** Cso. $T=5\ln\left(2+\dfrac{2}{e}\right)$ Condone *t* appearing for *T* throughout this question.
(c) **Alternative 1** 1st Mark (Method): $15e^{-0.2\times T}+\text{awrt }5.52e^{-0.2\times T}=7.5\Rightarrow e^{-0.2\times T}=\text{awrt }0.37$ 2nd Mark (Accuracy): $T=-5\ln\left(\text{awrt }0.37\right)$ or awrt 5.03 or $T=-5\ln\left(\dfrac{7.5}{\text{awrt }20.52}\right)$ **Alternative 2** 1st Mark (Method): $13.754e^{-0.2\times T}=7.5\Rightarrow T=-5\ln\left(\dfrac{7.5}{13.754}\right)$ or equivalent such as 3.03 2nd Mark (Accuracy): 3.03 + 2 = 5.03 Allow $-5\ln\left(\dfrac{7.5}{13.754}\right)+2$ **Alternative 3 (by trial and improvement)** 1st Mark (Method): $15e^{-0.2\times 5}+15e^{-0.2\times 10}=7.55$ or $15e^{-0.2\times 5.1}+15e^{-0.2\times 10.1}=7.40$ or any value between. 2nd Mark (Accuracy): Answer *T* =5.03.

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number

Candidate Number

Mathematics
International Advanced Level
Pure Mathematics P4

Sample Assessment Materials for first teaching September 2018
Time: 1 hour 30 minutes

Paper Reference
WMA14/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. Use the binomial series to find the expansion of

$$\frac{1}{(2+5x)^3} \qquad |x| < \frac{2}{5}$$

in ascending powers of x, up to and including the term in x^3

Give each coefficient as a fraction in its simplest form.

(6)

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Question 1 continued

Q1

(Total for Question 1 is 6 marks)

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2. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

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Question 2 continued

Q2

(Total for Question 2 is 7 marks)

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3. $$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C

(4)

(b) (i) Hence find $\int f(x)\,dx$

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form $a + \ln b$, where a and b are constants.

(6)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total for Question 3 is 10 marks)

4.

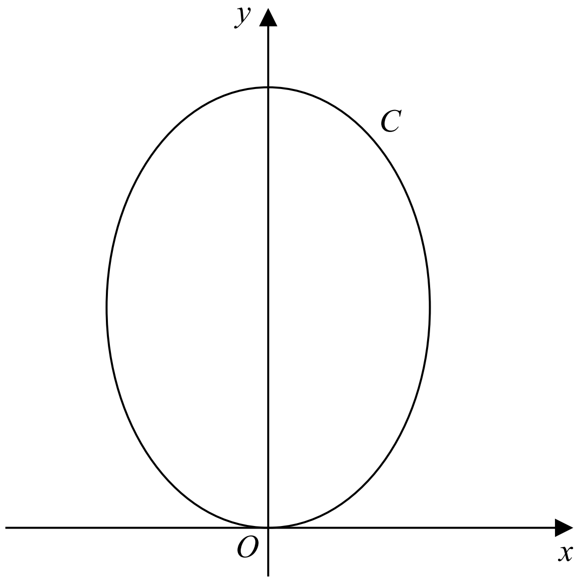

Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3}\sin 2t \qquad y = 4\cos^2 t \qquad 0 \leqslant t \leqslant \pi$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = k\sqrt{3}\tan 2t$, where k is a constant to be found.

(5)

(b) Find an equation of the tangent to C at the point where $t = \dfrac{\pi}{3}$

Give your answer in the form $y = ax + b$, where a and b are constants.

(4)

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Question 4 continued

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Question 4 continued

Q4

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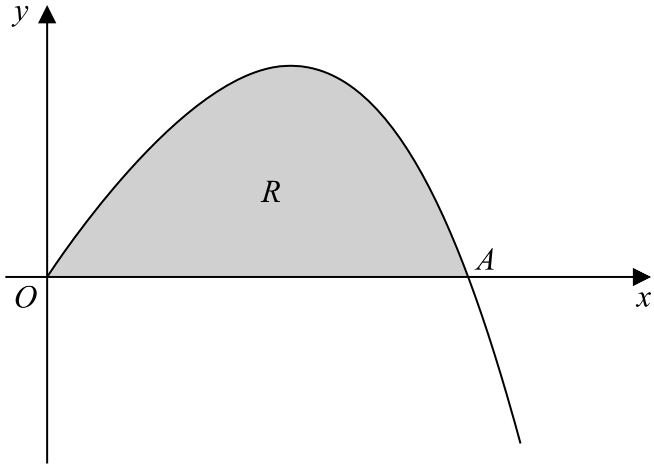

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of $\ln 2$, the x coordinate of the point A.

(2)

(b) Find $\int xe^{\frac{1}{2}x}\,dx$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

(c) Find, by integration, the exact value for the area of R.

Give your answer in terms of $\ln 2$

(3)

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Question 5 continued

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Question 5 continued

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Question 5 continued

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Q5

(Total for Question 5 is 8 marks)

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6. Prove by contradiction that, if a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$

(4)

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Question 6 continued

Q6

(Total for Question 6 is 4 marks)

7.

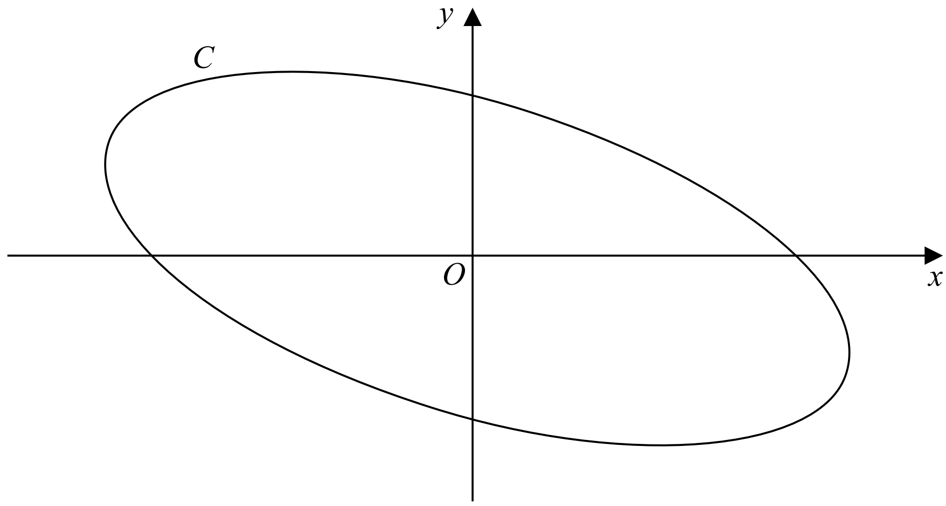

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right) \qquad y = 2\sin t \qquad 0 \leqslant t \leqslant 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be found.

(2)

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Question 7 continued

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Q7

(Total for Question 7 is 5 marks)

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8. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at time t is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \lambda(120 - \theta) \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100\mathrm{e}^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

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Question 8 continued

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Question 8 continued

Q8

(Total for Question 8 is 11 marks)

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9. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2

(2)

(c) Find the exact value of the distance AP.

Give your answer in the form $k\sqrt{2}$, where k is a constant to be found.

(2)

The acute angle between AP and l_2 is θ

(d) Find the value of $\cos\theta$

(3)

A point E lies on the line l_2

Given that $AP = PE$,

(e) find the area of triangle APE,

(2)

(f) find the coordinates of the two possible positions of E.

(5)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total for Question 9 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

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Pure Mathematics P4 Mark scheme

Question	Scheme	Marks
1	$\left\{\dfrac{1}{(2+5x)^3} = \right\} (2+5x)^{-3}$	M1
	$= \underline{(2)^{-3}}\left(1+\dfrac{5x}{2}\right)^{-3} = \underline{\dfrac{1}{8}}\left(1+\dfrac{5x}{2}\right)^{-3}$	B1
	$=\left\{\dfrac{1}{8}\right\}\left[1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3+\ldots\right]$ $=\left\{\dfrac{1}{8}\right\}\left[1+(-3)\left(\dfrac{5x}{2}\right)+\dfrac{(-3)(-4)}{2!}\left(\dfrac{5x}{2}\right)^2+\dfrac{(-3)(-4)(-5)}{3!}\left(\dfrac{5x}{2}\right)^3+\ldots\right]$ $=\dfrac{1}{8}\left[1-\dfrac{15}{2}x+\dfrac{75}{2}x^2-\dfrac{625}{4}x^3+\ldots\right]$ $=\dfrac{1}{8}[1-7.5x+37.5x^2-156.25x^3+\ldots]$	M1 A1
	$=\dfrac{1}{8}-\dfrac{15}{16}x;+\dfrac{75}{16}x^2-\dfrac{625}{32}x^3+\ldots$ **or** $\dfrac{1}{8}-\dfrac{15}{16}x;+4\dfrac{11}{16}x^2-19\dfrac{17}{32}x^3+\ldots$	A1 A1
		(6)

Notes:

M1: Mark can be implied by a constant term of $(2)^{-3}$ or $\dfrac{1}{8}$.

B1: $\underline{2^{-3}}$ or $\underline{\dfrac{1}{8}}$ outside brackets or $\dfrac{1}{8}$ as candidate's constant term in their binomial expansion.

M1: Expands $(\ldots+kx)^{-3}$, k = a value $\neq 1$ to give any 2 terms out of 4 terms simplified or un-simplified, Eg: $1+(-3)(kx)$ or $\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ or $1+\ldots+\dfrac{(-3)(-4)}{2!}(kx)^2$ or $\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ are fine for M1.

A1: A correct simplified or un-simplified $1+(-3)(kx)+\dfrac{(-3)(-4)}{2!}(kx)^2+\dfrac{(-3)(-4)(-5)}{3!}(kx)^3$ expansion with consistent (kx). Note that (kx) must be consistent and k = a value $\neq 1$. (on the RHS, not necessarily the LHS) in a candidate's expansion.

A1: For $\dfrac{1}{8}-\dfrac{15}{16}x$ **(simplified)** or also allow $0.125-0.9375x$.

A1: Accept only $\dfrac{75}{16}x^2-\dfrac{625}{32}x^3$ or $4\dfrac{11}{16}x^2-19\dfrac{17}{32}x^3$ or $4.6875x^2-19.53125x^3$

Question	Scheme	Marks
2(a)	$x^3 + 2xy - x - y^3 - 20 = 0$	
	$\left\{\cancel{\frac{\mathrm{d}y}{\mathrm{d}x}} \cancel{\times}\right\} \quad \underline{3x^2} + \underline{\underline{\left(2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x}\right)}} \underline{- 1 - 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$3x^2 + 2y - 1 + \left(2x - 3y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ **or** $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$ **cso**	A1
		(5)
(b)	At P(3, −2), $\mathrm{m}(\mathbf{T}) = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \frac{22}{6}$ or $\frac{11}{3}$ **and either T**: $y - -2 = \text{"}\frac{11}{3}\text{"}(x - 3)$ **or** $(-2) = \left(\frac{11}{3}\right)(3) + c \Rightarrow c = \ldots,$	M1
	T: $11x - 3y - 39 = 0$ **or** $K(11x - 3y - 39) = 0$ **cso**	A1
		(2)
		(7 marks)

Notes:

(a)

M1: Differentiates implicitly to include either $2y\frac{\mathrm{d}x}{\mathrm{d}y}$ or $x^3 \to \pm kx^2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-x \to -\frac{\mathrm{d}x}{\mathrm{d}y}$

(Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y} = \right)$).

A1: $x^3 \to 3x^2\frac{\mathrm{d}x}{\mathrm{d}y}$ **and** $-x - y^3 - 20 = 0 \to -\frac{\mathrm{d}x}{\mathrm{d}y} - 3y^2 = 0$

B1: $2xy \to 2y\frac{\mathrm{d}x}{\mathrm{d}y} + 2x$

dM1: **Dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}x}{\mathrm{d}y}$ as long as there are ***at least two terms*** in $\frac{\mathrm{d}x}{\mathrm{d}y}$.

A1: For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$

(b)

M1: **Some** attempt to substitute **both** $x = 3$ **and** $y = -2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which contains both x and y to find m_T **and**

- **either** applies $y - -2 = (\text{their } m_T)(x - 3)$, where m_T is a numerical value.
- **or** finds c by solving $(-2) = (\text{their } m_T)(3) + c$, where m_T is a numerical value.

A1: Accept any integer multiple of $11x - 3y - 39 = 0$ or $11x - 39 - 3y = 0$ or $-11x + 3y + 39 = 0$, where their tangent equation is equal to 0.

Question	Scheme	Marks
3(a)	$1 = A(3x-1)^2 + Bx(3x-1) + Cx$	B1
	$x \to 0 \quad (1 = A)$	M1
	$x \to \frac{1}{3} \quad 1 = \frac{1}{3}C \Rightarrow C = 3$ any two constants correct coefficients of x^2	A1
	$0 = 9A + 3B \Rightarrow B = -3$ all three constants correct	A1
		(4)
(b)(i)	$\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$ $= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$ $\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$	M1 A1ft A1ft
		(3)
(b)(ii)	$\int_1^2 f(x)\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$	
	$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$	M1
	$= \ln \frac{2 \times 2}{5} + \ ...$	M1
	$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$	A1
		(3)
		(10 marks)

Notes:

(a)

B1: Obtaining $1 = A(3x-1)^2 + Bx(3x-1) + Cx$ at any stage. This will usually be at the beginning of the solution but, if the cover-up rule is used, it could appear later.

M1: A complete method of finding any one of the three constants. If either $A = 1$ or $C = 3$ is given without working or, at least, without incorrect working, allow this M1 – use of the cover-up rule is acceptable. In principle, an alternative method is equating coefficients (or substituting three values other than 0 and $\frac{1}{3}$), obtaining a sufficient set of equations and solving for any one of the three constants.

A1: Any two of A, B and C correct. These will usually, but not always, be A and C.

A1: All three of A, B and C correct. If all three constants are correct and the answers do not clearly conflict with any working, allow all 4 marks (including the B1) bod. There are a number of possible ways of finding B but, as long as the M has been gained, you need not consider the method used.

Question 3 notes *continued*
(b)(ii) **M1:** Dependent upon the M mark in (b). Substituting in the correct limits and subtracting, not necessarily the right way round. There must be evidence that both 1 and 2 have been used but errors in substitution do not lose the mark. **M1:** Dependent upon both previous Ms. Applies the addition and/or subtraction rules of logs to obtain a single logarithm. Either the addition or the subtraction rule of logs must be used correctly at least once to gain this mark and this must be seen in the attempt at (b)(ii). **A1:** The correct answer in the form specified. Accept equivalent fractions including exact decimals for *a* and or *b*. Accept $\ln\frac{4}{5}+\frac{3}{10}$. $\frac{3}{10}-\ln\frac{5}{4}$ is not acceptable.

Question	Scheme	Marks
4(a)	$\dfrac{dx}{dt} = 2\sqrt{3}\cos 2t$	B1
	$\dfrac{dy}{dt} = -8\cos t\sin t$	M1 A1
	$\dfrac{dy}{dx} = \dfrac{-8\cos t\sin t}{2\sqrt{3}\cos 2t}$ $= -\dfrac{4\sin 2t}{2\sqrt{3}\cos 2t}$	M1
	$\dfrac{dy}{dx} = -\dfrac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\dfrac{2}{3}\right)$	A1
		(5)
(b)	When $t = \dfrac{\pi}{3}$ $\qquad x = \dfrac{3}{2},\ y = 1$ can be implied	B1
	$m = -\dfrac{2}{3}\sqrt{3}\tan\left(\dfrac{2\pi}{3}\right) \quad (=2)$	M1
	$y - 1 = 2\left(x - \dfrac{3}{2}\right)$	dM1
	$y = 2x - 2$	A1
		(4)
		(9 marks)

Notes:

(a)

B1: The correct $\dfrac{dx}{dt}$

M1: $\dfrac{dy}{dt} = \pm k\cos t\sin t$ or $\pm k\sin 2t$, where k is a non-zero constant. Allow $k = 1$

A1: $\dfrac{dy}{dt} = -8\cos t\sin t$ or $-4\sin 2t$ or equivalent. In this question, it is possible to get a correct answer after incorrect working, e.g. $2\cos 2t - 2 \to -4\sin 2t$. This should lose this mark and the next A but ignore in part (b).

M1: Their $\dfrac{dy}{dt}$ divided by their $\dfrac{dx}{dt}$, or their $\dfrac{dy}{dt}$ multiplied by their $\dfrac{dt}{dx}$. The answer must be a function of t only.

Question 4 notes *continued*	
A1:	The correct answer in the form specified. They don't have to explicitly state $k = -\frac{2}{3}$ but there must be evidence that the constant is $-\frac{2}{3}$. Accept equivalent fractions.
(b)	
B1:	That when $t = \frac{\pi}{3}$, $x = \frac{3}{2}$ and $y = 1$. Exact numerical values are required but the values can be implied, for example by a correct final answer, and can occur anywhere in the question.
M1:	Substituting $t = \frac{\pi}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$. Trigonometric terms, e.g. $\tan\frac{2\pi}{3}$ need not be evaluated.
dM1:	Dependent on the previous M. Finding an equation of a tangent with their point and their numerical value of the gradient of the tangent, not the normal. Expressions like $\tan\frac{2\pi}{3}$ must be evaluated. The equation must be linear. Using $y - y' = m(x - x')$. They should get x' and y' the right way round. Alternatively writing $y = (\text{their } m)x + c$ and using their point, the right way round, to find c.
A1:	cao. The correct answer in the form specified.

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="4">5(a)</td><td colspan="2">$y = 4x - xe^{\frac{1}{2}x}, x \geqslant 0$

$\left\{y = 0 \Rightarrow 4x - xe^{\frac{1}{2}x} = 0 \Rightarrow x(4 - e^{\frac{1}{2}x}) = 0 \Rightarrow\right\}$</td><td></td></tr>
<tr><td rowspan="2">$e^{\frac{1}{2}x} = 4 \Rightarrow x_A = 4\ln 2$</td><td>Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$</td><td>M1</td></tr>
<tr><td>$4\ln 2$ cao (Ignore $x = 0$)</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(2)</td></tr>
<tr><td rowspan="4">(b)</td><td rowspan="2">$\left\{\int xe^{\frac{1}{2}x}\,dx\right\} = 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$</td><td>$\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\}$, $\alpha > 0$, $\beta > 0$</td><td>M1</td></tr>
<tr><td>$2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\{dx\}$, with or without dx</td><td>A1</td></tr>
<tr><td colspan="2">$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\ \{+c\}$</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(3)</td></tr>
<tr><td rowspan="5">(c)</td><td colspan="2">$\left\{\int 4x\,dx\right\} = 2x^2$</td><td rowspan="2">B1</td></tr>
<tr><td colspan="2">$\left\{\int_0^{4\ln 2}(4x - xe^{\frac{1}{2}x})\,dx\right\} = \left[2x^2 - \left(2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}\right)\right]_0^{4\ln 2 \text{ or } \ln 16 \text{ or their limits}}$</td></tr>
<tr><td colspan="2">$= \left(2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 4e^{\frac{1}{2}(4\ln 2)}\right) - \left(2(0)^2 - 2(0)e^{\frac{1}{2}(0)} + 4e^{\frac{1}{2}(0)}\right)$</td><td>M1</td></tr>
<tr><td colspan="2">$= (32(\ln 2)^2 - 32(\ln 2) + 16) - (4)$
$= 32(\ln 2)^2 - 32(\ln 2) + 12$</td><td>A1</td></tr>
<tr><td colspan="2"></td><td>(3)</td></tr>
<tr><td colspan="4" align="right">(8 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">(a)
M1: Attempts to solve $e^{\frac{1}{2}x} = 4$ giving $x = ...$ in terms of $\pm\lambda\ln\mu$ where $\mu > 0$
A1: $4\ln 2$ cao stated in part (a) only (Ignore $x = 0$)</td></tr>
<tr><td colspan="4">(b)
M1: Integration by parts is applied in the form $\alpha xe^{\frac{1}{2}x} - \beta\int e^{\frac{1}{2}x}\{dx\}$, where $\alpha > 0, \beta > 0$.
(must be in this form) with or without dx</td></tr>
</table>

Question 5 notes *continued*

A1: $2x\mathrm{e}^{\frac{1}{2}x} - \int 2\mathrm{e}^{\frac{1}{2}x}\{\mathrm{d}x\}$ or equivalent, with or without $\mathrm{d}x$. **Can be un-simplified.**

A1: $2x\mathrm{e}^{\frac{1}{2}x} - 4\mathrm{e}^{\frac{1}{2}x}$ or equivalent with or without $+\,c$. **Can be un-simplified.**

(c)

B1: $4x \to 2x^2$ or $\dfrac{4x^2}{2}$ oe

M1: **Complete** method of applying limits of their x_A and 0 to all terms of an expression of the form $\pm Ax^2 \pm Bx\mathrm{e}^{\frac{1}{2}x} \pm C\mathrm{e}^{\frac{1}{2}x}$. (Where $A\,\square\,0$, $B\,\square\,0$ and $C\,\square\,0$) and subtracting the correct way round.

A1: A correct three term exact quadratic expression in $\ln 2$. For example allow for A1

- $32(\ln 2)^2 - 32(\ln 2) + 12$
- $8(2\ln 2)^2 - 8(4\ln 2) + 12$
- $2(4\ln 2)^2 - 32(\ln 2) + 12$
- $2(4\ln 2)^2 - 2(4\ln 2)\mathrm{e}^{\frac{1}{2}(4\ln 2)} + 12$

Note that the constant term of 12 needs to be combined from $4\mathrm{e}^{\frac{1}{2}(4\ln 2)} - 4\mathrm{e}^{\frac{1}{2}(0)}$ o.e.

Also allow $32\ln 2(\ln 2 - 1) + 12$ or $32\ln 2\left(\ln 2 - 1 + \dfrac{12}{32\ln 2}\right)$ for A1.

Allow $32(\ln^2 2) - 32(\ln 2) + 12$ for the final A1.

<table>
<tr><th>Question</th><th colspan="3">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="6">6</td><td colspan="3">Assumption: there exists positive real numbers a, b such that
$$a+b<2\sqrt{ab}$$</td><td>B1</td></tr>
<tr><td>Method 1</td><td>Method 2</td><td rowspan="2">A complete method for creating
$(f(a,b))^2 < 0$</td><td rowspan="2">M1A1</td></tr>
<tr><td>$a + b - 2\sqrt{ab} < 0$
$(\sqrt{a} - \sqrt{b})^2 < 0$</td><td>$(a + b)^2 = (2\sqrt{ab})^2$
$a^2 + 2ab + b^2 < 4ab$
$a^2 - 2ab + b^2 < 0$
$(a - b)^2 < 0$</td></tr>
<tr><td colspan="3">This is a contradiction, therefore</td><td rowspan="2">A1</td></tr>
<tr><td colspan="3">If a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$</td></tr>
<tr><td colspan="3"></td><td>(4)</td></tr>
<tr><td colspan="5" align="right">(4 marks)</td></tr>
<tr><td colspan="5">Notes:</td></tr>
<tr><td colspan="5">
B1: As this is proof by contradiction, the candidate is required to start their proof by assuming that the contrary. That is "if a, b are positive real numbers, then $a+b \geqslant 2\sqrt{ab}$" is true.
Accept, as a minimum, there exists a and b such that $a+b<2\sqrt{ab}$
M1: For starting with $a+b<2\sqrt{ab}$ and proceeding to either $(\sqrt{a}-\sqrt{b})^2 < 0$ or $(a-b)^2 < 0$
A1: All algebra is required to be correct. Do not accept, for instance, $(a + b)^2 = 2\sqrt{ab}^{\,2}$ even when followed by correct lines.
A1: A fully correct proof by contradiction. It must include a statement that $(a-b)^2 < 0$ is a contradiction so if a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$
</td></tr>
</table>

Question	Scheme		Marks
7(a)	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$		M1
	So, $\{x + y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ * **cso**		A1*
			(3)
(b)	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$		
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$\Rightarrow (x+y)^2 + 3y^2 = 12$	A1
		$\{a = 3, b = 12\}$	**(2)**
	Alternative		
	$(x+y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	$(x+y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x+y)^2 + 3y^2 = 12$	$(x+y)^2 + 3y^2 = 12$	A1
			(2)
			(5 marks)

Notes:

(a)

M1: $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t + \frac{\pi}{6}\right) \rightarrow \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$

dM1: Adds their expanded x (which is in terms of t) to $2\sin t$.

A1*: Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.

(b)

M1: Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.

A1: leading $(x+y)^2 + 3y^2 = 12$

Question	Scheme		Marks
8(a)	$\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \lambda(120 - \theta),\ \theta \leqslant 100$		
	$\displaystyle\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$		B1
	$-\ln(120 - \theta); = \lambda t + c$	For integrating lhs M1 A1 For integrating rhs M1 A1	M1A1; M1A1
	$\{t = 0, \theta = 20 \ \Rightarrow\}\ -\ln(100) = \lambda(0) + c$ $\Rightarrow -\ln(120 - \theta) = \lambda t - \ln 100$ $\Rightarrow\ -\lambda t = \ln(120 - \theta) - \ln 100$ $\Rightarrow\ -\lambda t = \ln\left(\dfrac{120-\theta}{100}\right)$		M1
	$\mathrm{e}^{-\lambda t} = \dfrac{120-\theta}{100}$		dddM1
	$100\,\mathrm{e}^{-\lambda t} = 120 - \theta$ leading to $\theta = 120 - 100\mathrm{e}^{-\lambda t}$		A1*
			(8)
(b)	$\{\lambda = 0.01, \theta = 100 \Rightarrow\}$ $\quad 100 = 120 - 100\,\mathrm{e}^{-0.01t}$		M1
	$\Rightarrow 100\,\mathrm{e}^{-0.01t} = 120 - 100 \Rightarrow$ $-0.01t = \ln\left(\dfrac{120-100}{100}\right)$ $t = \dfrac{1}{-0.01}\ln\left(\dfrac{120-100}{100}\right)$ $\left\{t = \dfrac{1}{-0.01}\ln\left(\dfrac{1}{5}\right) = 100\ln 5\right\}$	Uses correct order of operations by moving from $100 = 120 - 100\mathrm{e}^{-0.01t}$ to give $t = ...$ and $t = A\ln B$, where $B > 0$	dM1
	$t = 160.94379\ldots$ 161 (s) (nearest second) awrt 161		A1
			(3)
			(11 marks)

Notes:

(a)

B1M1A1M1A1: Mark as in the scheme.

M1: Substitutes $t = 0$ AND $\theta = 20$ in an integrated equation leading to $\pm\lambda t = \ln(\mathrm{f}(\theta))$

dddM1: Uses a fully correct method to eliminate their logarithms and writes down an equation containing their evaluated constant of integration.

A1*: Correct answer with no errors. This is a given answer

(b)

M1: Substitutes $\lambda = 0.01$, $\theta = 100$ into given equation

M1: See scheme

A1: Awrt 161 seconds.

Question	Scheme		Marks
9 (a)	$A(3, 5, 0)$		B1
			(1)
(b)	$\{l_2 :\}\ \mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$	$\mathbf{a} + \lambda\mathbf{d}$ or $\mathbf{a} + \mu\mathbf{d}$, $\mathbf{a} + t\mathbf{d}$ $\mathbf{a} \neq 0, \mathbf{d} \neq 0$ with either $\mathbf{a} = \mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ or $\mathbf{d} = -5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, or a multiple of $-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$	M1
		Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$	A1
			(2)
(c)	$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \begin{pmatrix}1\\5\\2\end{pmatrix} - \begin{pmatrix}3\\5\\0\end{pmatrix} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$		
	$AP = \sqrt{(-2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$	Full method for finding AP	M1
	$2\sqrt{2}$		A1
			(2)
(d)	So $\overrightarrow{AP} = \begin{pmatrix}-2\\0\\2\end{pmatrix}$ and $\mathbf{d}_2 = \begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow \begin{pmatrix}-2\\0\\2\end{pmatrix}.\begin{pmatrix}-5\\4\\3\end{pmatrix}$	Realisation that the dot product is required between $\left(\overrightarrow{AP}\text{ or }\overrightarrow{PA}\right)$ and $\pm K\mathbf{d}_2$ or $\pm K\mathbf{d}_1$	M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AP}\,.\,\mathbf{d}_2}{\lvert\overrightarrow{AP}\rvert\,\lvert\mathbf{d}_2\rvert} = \dfrac{\pm\left(\begin{pmatrix}-2\\0\\2\end{pmatrix}.\begin{pmatrix}-5\\4\\3\end{pmatrix}\right)}{\sqrt{(-2)^2 + (0)^2 + (2)^2}\,.\sqrt{(-5)^2 + (4)^2 + (3)^2}}$		dM1
	$\{\cos\theta\} = \dfrac{\pm(10+0+6)}{\sqrt{8}.\sqrt{50}} = \underline{\dfrac{4}{5}}$		A1 cso
			(3)
(e)	$\{\text{Area } APE =\}\ \dfrac{1}{2}(\text{their } 2\sqrt{2})^2\sin\theta$		M1
	$= 2.4$		A1
			(2)

Question	Scheme		Marks
9(f)	$\overrightarrow{PE} = (-5\lambda)\mathbf{i} + (4\lambda)\mathbf{j} + (3\lambda)\mathbf{k}$ and $PE =$ their $2\sqrt{2}$ from part (c)		
	$\{PE^2 =\}\ (-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})^2$	This mark can be implied.	M1
	$\left\{\Rightarrow 50\lambda^2 = 8 \Rightarrow \lambda^2 = \frac{4}{25} \Rightarrow\right\}\ \lambda = \pm\frac{2}{5}$	Either $\lambda = \frac{2}{5}$ or $\lambda = -\frac{2}{5}$	A1
	$l_2 : \mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} \pm \frac{2}{5}\begin{pmatrix}-5\\4\\3\end{pmatrix}$	**dependent on the previous M mark** Substitutes at least one of their values of λ into l_2.	dM1
	$\{\overrightarrow{OE}\} = \begin{pmatrix}3\\\frac{17}{5}\\\frac{4}{5}\end{pmatrix}$ or $\begin{pmatrix}3\\3.4\\0.8\end{pmatrix}$, $\{\overrightarrow{OE}\} = \begin{pmatrix}-1\\\frac{33}{5}\\\frac{16}{5}\end{pmatrix}$ or $\begin{pmatrix}-1\\6.6\\3.2\end{pmatrix}$	At least one set of coordinates are correct.	A1
		Both sets of coordinates are correct.	A1
			(5)
			(15 marks)

Notes:

(a)

B1: Allow $A(3, 5, 0)$ or $3\mathbf{i} + 5\mathbf{j}$ or $3\mathbf{i} + 5\mathbf{j} + 0\mathbf{k}$ or $\begin{pmatrix}3\\5\\0\end{pmatrix}$ or benefit of the doubt $\begin{matrix}3\\5\\0\end{matrix}$

(b)

A1: Correct vector equation using $\mathbf{r} =$ **or** $l =$ **or** $l_2 =$ **or** Line 2 =

i.e. Writing $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}-5\\4\\3\end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix}1\\5\\2\end{pmatrix} + \lambda\mathbf{d}$, where $\mathbf{d}$ is a multiple of $\begin{pmatrix}-5\\4\\3\end{pmatrix}$.

Note: Allow the use of parameters μ or t instead of λ.

(c)

M1: Finds the difference between $\overrightarrow{OP}$ and their $\overrightarrow{OA}$ and applies Pythagoras to the result to find AP

Note: Allow M1A1 for $\begin{pmatrix}2\\0\\2\end{pmatrix}$ leading to $AP = \sqrt{(2)^2 + (0)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$.

Question 9 notes *continued*
(d) **M1:** Realisation that the dot product is required between $\left(\overrightarrow{AP} \text{ or } \overrightarrow{PA}\right)$ **dM1:** Full method to find $\cos\theta$ (dependent upon the previous M), **A1:** $\cos\theta = \frac{4}{5}$ or exact equivalent
(e) **M1 A1:** For $\frac{1}{2}(2\sqrt{2})^2\sin(36.869...^\circ)$ or $\frac{1}{2}(2\sqrt{2})^2\sin(180^\circ - 36.869...^\circ)$; = awrt 2.40 Candidates must use their θ from part (d) or apply a correct method of finding their $\sin\theta = \frac{3}{5}$ from their $\cos\theta = \frac{4}{5}$
(f) **M1:** Allow special case 1st M1 for $\lambda = 2.5$ from comparing lengths or from no working. for $\sqrt{(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2} = (\text{their } 2\sqrt{2})$ 1st M0 for $(-5\lambda)^2 + (4\lambda)^2 + (3\lambda)^2 = (\text{their } 2\sqrt{2})$ or equivalent. 1st M1 for $\lambda = \frac{\text{their } AP = \text{"}2\sqrt{2}\text{"}}{\sqrt{(-5)^2 + (4)^2 + (3)^2}}$ and 1st A1 for $\lambda = \frac{2\sqrt{2}}{5\sqrt{2}}$ So $\left\{\mathbf{d}_1 = \frac{1}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix} \Rightarrow\right\}$ $\text{"vector"} = \frac{2\sqrt{2}}{5\sqrt{2}}\begin{pmatrix}-5\\4\\3\end{pmatrix}$ is M1A1 **dM1:** In part (f) can be implied for at least 2 (out of 6) correct *x*, *y*, *z* ordinates from their values of λ.

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number

Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics FP1

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WFM11/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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S 5 9 7 5 9 A 0 1 3 2

Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{n}{4}(n + a)(n + b)(n + c)$$

where a, b and c are integers to be found.

(4)

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Question 1 continued

Q1

(Total for Question 1 is 4 marks)

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2. A parabola P has cartesian equation $y^2 = 28x$. The point S is the focus of the parabola P.

(a) Write down the coordinates of the point S.

(1)

Points A and B lie on the parabola P. The line AB is parallel to the directrix of P and cuts the x-axis at the midpoint of OS, where O is the origin.

(b) Find the exact area of triangle ABS.

(4)

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Question 2 continued

Q2

(Total for Question 2 is 5 marks)

3.

$$f(x) = x^2 + \frac{3}{x} - 1, \qquad x < 0$$

The only real root, α, of the equation $f(x) = 0$ lies in the interval $[-2, -1]$.

(a) Taking -1.5 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 2 decimal places.

(5)

(b) Show that your answer to part (a) gives α correct to 2 decimal places.

(2)

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Question 3 continued

Q3

(Total for Question 3 is 7 marks)

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4. Given that

$$\mathbf{A} = \begin{pmatrix} k & 3 \\ -1 & k+2 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) show that $\det(\mathbf{A}) > 0$ for all real values of k,

(3)

(b) find $\mathbf{A}^{-1}$ in terms of k.

(2)

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Question 4 continued

Q4

(Total for Question 4 is 5 marks)

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5.

$$2z + z^* = \frac{3 + 4\text{i}}{7 + \text{i}}$$

Find z, giving your answer in the form $a + b\text{i}$, where a and b are real constants. You must show all your working.

(5)

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Question 5 continued

Q5

(Total for Question 5 is 5 marks)

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6. The rectangular hyperbola H has equation $xy = 25$

(a) Verify that, for $t \neq 0$, the point $P\left(5t, \dfrac{5}{t}\right)$ is a general point on H.

(1)

The point A on H has parameter $t = \dfrac{1}{2}$

(b) Show that the normal to H at the point A has equation

$$8y - 2x - 75 = 0$$

(5)

This normal at A meets H again at the point B.

(c) Find the coordinates of B.

(4)

Leave blank

Question 6 continued

Question 6 continued

Leave blank

Question 6 continued

Q6

(Total for Question 6 is 10 marks)

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7.

$$\mathbf{P} = \begin{pmatrix} \dfrac{5}{13} & -\dfrac{12}{13} \\ \dfrac{12}{13} & \dfrac{5}{13} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix $\mathbf{P}$.
(3)

The transformation V, represented by the 2×2 matrix $\mathbf{Q}$, is a reflection in the line with equation $y = x$

(b) Write down the matrix $\mathbf{Q}$.
(1)

Given that the transformation V followed by the transformation U is the transformation T, which is represented by the matrix $\mathbf{R}$,

(c) find the matrix $\mathbf{R}$.
(2)

(d) Show that there is a value of k for which the transformation T maps each point on the straight line $y = kx$ onto itself, and state the value of k.
(4)

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Question 7 continued

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Question 7 continued

Q7

(Total for Question 7 is 10 marks)

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8.

$$f(z) = z^4 + 6z^3 + 76z^2 + az + b$$

where a and b are real constants.

Given that $-3 + 8i$ is a complex root of the equation $f(z) = 0$

(a) write down another complex root of this equation. **(1)**

(b) Hence, or otherwise, find the other roots of the equation $f(z) = 0$ **(6)**

(c) Show on a single Argand diagram all four roots of the equation $f(z) = 0$ **(2)**

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total for Question 8 is 9 marks)

9. The quadratic equation

$$2x^2 + 4x - 3 = 0$$

has roots α and β.

Without solving the quadratic equation,

(a) find the exact value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

(5)

(b) Find a quadratic equation which has roots $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(4)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total for Question 9 is 9 marks)

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10. (i) A sequence of positive numbers is defined by

$$u_1 = 5$$
$$u_{n+1} = 3u_n + 2, \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 2 \times (3)^n - 1$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3 + 2n)}{3^n}$$

(6)

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Question 10 continued

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Q10

(Total for Question 10 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

Further Pure Mathematics FP1 Mark scheme

Question	Scheme		Marks
1	$\sum_{r=1}^{n} r(r^2-3) = \sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r$		
	$= \frac{1}{4}n^2(n+1)^2 - 3\left(\frac{1}{2}n(n+1)\right)$	Attempts to expand $r(r^2-3)$ and attempts to substitute at least one correct standard formula into their resulting expression.	M1
		Correct expression (or equivalent)	A1
	$= \frac{1}{4}n(n+1)\left[n(n+1)-6\right]$	**dependent on the previous M mark** Attempt to factorise at least $n(n+1)$ having attempted to substitute both the standard formulae	dM1
	$= \frac{1}{4}n(n+1)\left[n^2+n-6\right]$	{this step does not have to be written]	
	$= \frac{1}{4}n(n+1)(n+3)(n-2)$	Correct completion with no errors	A1 cso
			(4)
			(4 marks)

Notes:

Applying eg. $n=1, n=2, n=3$ to the printed equation without applying the standard formulae to give $a=1, b=3, c=-2$ or another combination of these numbers is M0A0M0A0.

<u>**Alternative Method**</u>:

Obtains $\sum_{r=1}^{n} r(r^2-3) \equiv \frac{1}{4}n(n+1)\left[n(n+1)-6\right] \equiv \frac{1}{4}n(n+a)(n+b)(n+c)$

So $a=1$. $n=1 \Rightarrow -2 = \frac{1}{4}(1)(2)(1+b)(1+c)$ and $n=2 \Rightarrow 0 = \frac{1}{4}(2)(3)(2+b)(2+c)$

leading to either $b=-2, c=3$ or $b=3, c=-2$

dM1: dependent on the previous M mark.
Substitutes in values of n and solves to find $b=...$ and $c=...$

A1: Finds $a=1, b=3, c=-2$ or another combination of these numbers.

Using **only** a method of "proof by induction" scores 0 marks unless there is use of the standard formulae when the first M1 may be scored.

Allow final dM1A1 for $\frac{1}{4}n^4+\frac{1}{2}n^3-\frac{5}{4}n^2-\frac{3}{2}n$ or $\frac{1}{4}n(n^3+2n^2-5n-6)$

or $\frac{1}{4}(n^4+2n^3-5n^2-6n) \rightarrow \frac{1}{4}n(n+1)(n+3)(n-2)$, from no incorrect working.

Give final A0 for eg. $\frac{1}{4}n(n+1)\left[n^2+n-6\right] \rightarrow = \frac{1}{4}n(n+1)(x+3)(x-2)$ unless recovered.

Question	Scheme		Marks
2(a)	$P: y^2 = 28x$ or $P(7t^2, 14t)$		
	$(y^2 = 4ax \Rightarrow a = 7) \Rightarrow S(7,0)$	Accept $(7,0)$ or $x = 7,\ y = 0$ or 7 marked on the x-axis in a sketch	B1
			(1)
(b)	$\{A$ and B have x coordinate$\}$ $\frac{7}{2}$	Divides their x coordinate from (a) by 2 **and** substitutes this into the parabola equation and takes the sqaure root to find $y = ...$ **or applies** $y = \sqrt{\left(2(''7'') - \left(\frac{''7''}{2}\right)\right)^2 - \left(\frac{''7''}{2}\right)^2}$ **or solves** $7t^2 = 3.5$ and finds $y = 2(7)''\text{their } t''$	M1
	So $y^2 = 28\left(\frac{7}{2}\right) \Rightarrow y^2 = 98 \Rightarrow y = ...$ **or** $y = \sqrt{(2(7) - 3.5)^2 - (3.5)^2}$ $\left\{= \sqrt{(10.5)^2 - (3.5)^2}\right\}$ **or** $7t^2 = 3.5 \Rightarrow t = \sqrt{0.5} \Rightarrow y = 2(7)\sqrt{0.5}$		
	$y = (\pm)7\sqrt{2}$	***At least one*** correct **exact** value of y. Can be un-simplified or simplified.	A1
	A, B have coordinates $\left(\frac{7}{2}, 7\sqrt{2}\right)$ and $\left(\frac{7}{2}, -7\sqrt{2}\right)$		
	Area triangle $ABS =$		
	• $\frac{1}{2}\left(2(7\sqrt{2})\right)\left(\frac{7}{2}\right)$ • $\frac{1}{2}\begin{vmatrix} 7 & 3.5 & 3.5 & 7 \\ 0 & 7\sqrt{2} & -7\sqrt{2} & 0 \end{vmatrix}$	**dependent on the previous M mark** A **full** method for finding the area of triangle ABS.	dM1
	$= \frac{49}{2}\sqrt{2}$	Correct exact answer.	A1
			(4)
			(5 marks)

Question 2 *continued*
Notes:
(a) You can give B1 for part (a) for correct relevant work seen in either part (a) or part (b).
(b) **1st M1:** Allow a slip when candidates find the x coordinate of their midpoint as long as $0 <$ their midpoint $<$ their a Give 1st M0 if a candidate finds and uses $y = 98$ **1st A1:** Allow any **exact value** of either $7\sqrt{2}, -7\sqrt{2}, \sqrt{98}, -\sqrt{98}, 14\sqrt{0.5}$, awrt 9.9 or awrt -9.9 **2nd dM1:** Either $\frac{1}{2}\left(2\times \text{their "}7\sqrt{2}\text{"}\right)\left(\text{their } x_{\text{midpoint}}\right)$ or $\frac{1}{2}\left(2\times \text{their "}7\sqrt{2}\text{"}\right)\left(\text{their "7"} - x_{\text{midpoint}}\right)$ Condone area triangle $ABS = \left(7\sqrt{2}\right)\left(\frac{7}{2}\right)$, i.e. $\left(\text{their "}7\sqrt{2}\text{"}\right)\left(\frac{\text{their "7"}}{2}\right)$ **2nd A1:** Allow exact answers such as $\frac{49}{2}\sqrt{2}, \frac{49}{\sqrt{2}}, 24.5\sqrt{2}, \frac{\sqrt{4802}}{2}, \sqrt{\frac{4802}{4}}, 3.5\sqrt{2}, 49\sqrt{\frac{1}{2}}$ or $\frac{7}{2}\sqrt{98}$ but do not allow $\frac{1}{2}(3.5)\left(2\sqrt{98}\right)$ seen by itself. Give final A0 for finding 34.64823228… without reference to a correct exact value.

Question	Scheme		Marks
3(a)	$\mathrm{f}(x) = x^2 + \frac{3}{x} - 1, \quad x < 0$		
	$\mathrm{f}'(x) = 2x - 3x^{-2}$	At one of either $x^2 \to \pm Ax$ or $\frac{3}{x} \to \pm Bx^{-2}$ where A and B are non-zero constants.	M1
		Correct differentiation	A1
	$\mathrm{f}(-1.5) = -0.75$, $\mathrm{f}'(-1.5) = -\frac{13}{3}$	Either $\mathrm{f}(-1.5) = -0.75$ or $\mathrm{f}'(-1.5) = -\frac{13}{3}$ or awrt -4.33 **or a correct numerical expression** for either $\mathrm{f}(-1.5)$ or $\mathrm{f}'(-1.5)$ **Can be implied by later working**	B1
	$\left\{\alpha \simeq -1.5 - \frac{\mathrm{f}(-1.5)}{\mathrm{f}'(-1.5)}\right\} \Rightarrow \alpha \simeq -1.5 - \frac{-0.75}{-4.333333...}$	**dependent on the previous M mark** Valid attempt at Newton-Raphson using their values of $\mathrm{f}(-1.5)$ and $\mathrm{f}'(-1.5)$	dM1
	$\left\{\alpha = -1.67307692...\text{ or } -\frac{87}{52}\right\} \Rightarrow \alpha = -1.67$	**dependent on all 4 previous marks** -1.67 on their first iteration (Ignore any subsequent iterations)	A1 cso cao
	Correct differentiation followed by a correct answer scores full marks in (a) **Correct answer with <u>no</u> working scores no marks in (a)**		
			(5)
(b)	**Way 1**		
	$\mathrm{f}(-1.675) = \quad 0.01458022...$ $\mathrm{f}(-1.665) = -0.0295768...$	Chooses a suitable interval for x, which is within ± 0.005 of their answer to (a) and at least one attempt to evaluate $\mathrm{f}(x)$.	M1
	Sign change (positive, negative) (and $\mathrm{f}(x)$ is continuous) therefore (a root) $\alpha = -1.67$ (2 dp)	Both values correct awrt (or truncated) 1 sf, sign change and conclusion.	A1 cso
			(2)

Question	Scheme		Marks
3(b) *continued*	**Way 2**		
	Alt 1: Applying Newton-Raphson again Eg. Using $\alpha = -1.67, -1.673$ or $-\frac{87}{52}$		
	• $\alpha \simeq -1.67 - \frac{-0.007507185629...}{-4.415692926...}$ $\{= -1.671700115...\}$ • $\alpha \simeq -1.673 - \frac{0.005743106396...}{-4.41783855...}$ $\{= -1.671700019...\}$ • $\alpha \simeq -\frac{87}{52} - \frac{0.006082942257...}{-4.417893838...}$ $\{= -1.67170036...\}$	Evidence of applying Newton-Raphson for a second time on their answer to part (a)	M1
	So $\alpha = -1.67$ (2 dp)	$\alpha = -1.67$	A1
			(2)
			(7 marks)

Notes:

(a)

Incorrect differentiation followed by their estimate of α with no evidence of applying the NR formula is final dM0A0.

B1: B1 can be given for a correct numerical expression for either $f(-1.5)$ or $f'(-1.5)$

Eg. either $(-1.5)^2 + \frac{3}{(-1.5)} - 1$ or $2(-1.5) - \frac{3}{(-1.5)^2}$ are fine for B1.

Final -This mark can be implied by applying at least one correct value of either $f(-1.5)$ or $f'(-1.5)$

dM1: in $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$. So just $-1.5 - \frac{f(-1.5)}{f'(-1.5)}$ with an incorrect answer and no other evidence scores final dM0A0.

Give final dM0 for applying $1.5 - \frac{f(-1.5)}{f'(-1.5)}$ without first quoting the correct N-R formula.

(b)

A1: **Way 1:** correct solution only

Candidate needs to state both of their values for $f(x)$ to awrt (or truncated) 1 sf along with a reason and conclusion. Reference to change of sign or eg. $f(-1.675) \times f(-1.665) < 0$ or a diagram or < 0 and > 0 or one positive, one negative are sufficient reasons. There must be a (minimal, not incorrect) conclusion, eg. $\alpha = -1.67$, root (or α or part (a)) is correct, QED and a square are all acceptable. Ignore the presence or absence of any reference to continuity.

A minimal acceptable reason and conclusion is "change of sign, hence root".

No explicit reference to 2 decimal places is required.

Stating "root is in between – 1.675 and – 1.665" without some reference to is not sufficient for A1

Accept 0.015 as a correct evaluation of $f(-1.675)$

Question 3 notes *continued*
(b) **A1:** **Way 2:** correct solution only Their conclusion in Way 2 needs to convey that they understand that $\alpha = -1.67$ to 2 decimal places. Eg. "therefore my answer to part (a) [which must be -1.67] is correct" is fine for A1. $-1.67 - \frac{f(-1.67)}{f'(1.67)} = -1.67$ (2 dp) is sufficient for M1A1 in part (b). The root of $f(x) = 0$ is $-1.67169988...$, so candidates can also choose x_1 which is less than $-1.67169988...$ and choose x_2 which is greater than $-1.67169988...$ with both x_1 and x_2 lying in the interval $[-1.675, -1.665]$ and evaluate $f(x_1)$ and $f(x_2)$.
<u>Helpful Table</u>

x	$f(x)$
-1.675	0.014580224
-1.674	0.010161305
-1.673	0.005743106
-1.672	0.001325627
-1.671	-0.003091136
-1.670	-0.007507186
-1.669	-0.011922523
-1.668	-0.016337151
-1.667	-0.020751072
-1.666	-0.025164288
-1.665	-0.029576802

Question	Scheme		Marks
4(a)	$\mathbf{A} = \begin{pmatrix} k & 3 \\ -1 & k+2 \end{pmatrix}$ where k is a constant and let $g(k) = k^2 + 2k + 3$		
	$\{\det(\mathbf{A}) =\}\ k(k+2)+3$ or k^2+2k+3	Correct $\det(\mathbf{A})$, un-simplified or simplified	B1
	Way 1		
	$= (k+1)^2 - 1 + 3$	Attempts to complete the square [usual rules apply]	M1
	$= (k+1)^2 + 2 > 0$	$(k+1)^2 + 2$ and > 0	A1 cso
			(3)
	Way 2		
	$\{\det(\mathbf{A}) =\}\ k(k+2)+3$ or k^2+2k+3	Correct $\det(\mathbf{A})$, un-simplified or simplified	B1
	$\{b^2 - 4ac =\}\ 2^2 - 4(1)(3)$	Applies "$b^2 - 4ac$" to their $\det(\mathbf{A})$	M1
	All of • $b^2 - 4ac = -8 < 0$ • some reference to k^2+2k+3 being above the x-axis • so $\det(\mathbf{A}) > 0$	Complete solution	A1 cso
			(3)
	Way 3		
	$\{g(k) = \det(\mathbf{A}) =\}\ k(k+2)+3$ or k^2+2k+3	Correct $\det(\mathbf{A})$, un-simplified or simplified	B1
	$g'(k) = 2k+2 = 0 \Rightarrow k = -1$ $g_{\min} = (-1)^2 + 2(-1) + 3$	Finds the value of k for which $g'(k) = 0$ **and** substitutes this value of k into $g(k)$	M1
	$g_{\min} = 2$, so $\det(\mathbf{A}) > 0$	$g_{\min} = 2$ and states $\det(\mathbf{A}) > 0$	A1 cso
			(3)
(b)	$\mathbf{A}^{-1} = \dfrac{1}{k^2+2k+3}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$	$\dfrac{1}{\text{their } \det(\mathbf{A})}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$	M1
		Correct answer in terms of k	A1
			(2)
			(5 marks)

Question 4 *continued*

Notes:

(a)

B1: Also allow $k(k+2)--3$

Way 2: Proving $b^2-4ac=-8<0$ by itself could mean that $\det(\mathbf{A})>0$ or $\det(\mathbf{A})<0$.

To gain the final A1 mark for Way 2, candidates need to show $b^2-4ac=-8<0$ **and** make some reference to k^2+2k+3 being above the x-axis (eg. states that coefficient of k^2 is positive **or** evaluates $\det(\mathbf{A})$ for any value of k to give a positive result **or** sketches a quadratic curve that is above the x-axis) before then stating that $\det(\mathbf{A})>0$.

Attempting to solve $\det(\mathbf{A})=0$ by applying the quadratic formula or finding $-1\pm\sqrt{2}\mathrm{i}$ is enough to score the M1 mark for Way 2. To gain A1 these candidates need to make some reference to k^2+2k+3 being above the x-axis (eg. states that coefficient of k^2 is positive **or** evaluates $\det(\mathbf{A})$ for any value of k to give a positive result **or** sketches a quadratic curve that is above the x-axis) before then stating that $\det(\mathbf{A})>0$.

(b)

A1: Allow either $\dfrac{1}{(k+1)^2+2}\begin{pmatrix} k+2 & -3 \\ 1 & k \end{pmatrix}$ or $\begin{pmatrix} \dfrac{k+2}{k^2+2k+3} & \dfrac{-3}{k^2+2k+3} \\ \dfrac{1}{k^2+2k+3} & \dfrac{k}{k^2+2k+3} \end{pmatrix}$ or equivalent.

Question	Scheme		Marks
5	$2z + z^* = \dfrac{3+4\text{i}}{7+\text{i}}$		
	Way 1		
	$\left\{2z + z^* =\right\} 2(a+\text{i}b) + (a-\text{i}b)$	Left hand side $= 2(a+\text{i}b) + (a-\text{i}b)$ Can be implied by eg. $3a + \text{i}b$ **Note:** This can be seen anywhere in their solution	B1
	$\text{..........} = \dfrac{(3+4\text{i})}{(7+\text{i})}\dfrac{(7-\text{i})}{(7-\text{i})}$	Multiplies numerator **and** denominator of the right hand side by $7-\text{i}$ or $-7+\text{i}$	M1
	$\text{..........} = \dfrac{25+25\text{i}}{50}$	Applies $\text{i}^2 = -1$ to and collects like terms to give right hand side $= \dfrac{25+25\text{i}}{50}$ or equivalent	A1
	So, $3a + \text{i}b = \dfrac{1}{2} + \dfrac{1}{2}\text{i}$ $\Rightarrow a = \dfrac{1}{6}, b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$	**dependent on the previous B and M marks** Equates either real parts or imaginary parts to give at least one of $a = ...$ or $b = ...$	ddM1
		Either $a = \dfrac{1}{6}$ **and** $b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$	A1
			(5)
	Way 2		
	$\left\{2z + z^* =\right\} 2(a+\text{i}b) + (a-\text{i}b)$	Left hand side $= 2(a+\text{i}b) + (a-\text{i}b)$ Can be implied by eg. $3a + \text{i}b$	B1
	$(3a + \text{i}b)(7+\text{i}) = \text{..........}$	Multiplies their $(3a + \text{i}b)$ by $(7+\text{i})$	M1
	$21a + 3a\text{i} + 7b\text{i} - b = \text{..........}$	Applies $\text{i}^2 = -1$ to give left hand side $= 21a + 3a\text{i} + 7b\text{i} - b$	A1
	So, $(21a - b) + (3a + 7b) = 3 + 4\text{i}$ gives $21a - b = 3$, $3a + 7b = 4$ $\Rightarrow a = \dfrac{1}{6}, b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$	**dependent on the previous B and M marks** Equates **both** real parts and imaginary parts to give at least one of $a = ...$ or $b = ...$	ddM1
		Either $a = \dfrac{1}{6}$ **and** $b = \dfrac{1}{2}$ **or** $z = \dfrac{1}{6} + \dfrac{1}{2}\text{i}$	A1
			(5)
			(5 marks)

Question 5 *continued*
Notes:
Some candidates may let $z = x + \mathrm{i}y$ and $z^* = x - \mathrm{i}y$. So apply the mark scheme with $x \equiv a$ and $y \equiv b$. For the final A1 mark, you can accept exact equivalents for a, b.

Question	Scheme		Marks
6(a)	$H: xy = 25$, $P\left(5t, \dfrac{5}{t}\right)$ is a general point on H		
	Either $5t\left(\dfrac{5}{t}\right) = 25$ **or** $y = \dfrac{25}{x} = \dfrac{25}{5t} = \dfrac{5}{t}$ **or** $x = \dfrac{25}{y} = \dfrac{25}{\frac{5}{t}} = 5t$ **or** states $c = 5$		B1
			(1)
(b)	$y = \dfrac{25}{x} = 25x^{-1} \Rightarrow \dfrac{dy}{dx} = -25x^{-2} = -\dfrac{25}{x^2}$	$\dfrac{dy}{dx} = \pm k x^{-2}$ where k is a numerical value	M1
	$xy = 25 \Rightarrow x\dfrac{dy}{dx} + y = 0$	Correct use of product rule. The sum of two terms, one of which is correct.	
	$\dfrac{dy}{dx} = \dfrac{dy}{dt}.\dfrac{dt}{dx} = -\dfrac{5}{t^2}\left(\dfrac{1}{5}\right)$	$\dfrac{dy}{dt} \times \dfrac{1}{\text{their } \dfrac{dx}{dt}}$	
	$\left\{\text{At } A,\ t = \dfrac{1}{2},\ x = \dfrac{5}{2},\ y = 10\right\} \Rightarrow \dfrac{dy}{dx} = -4$	Correct numerical gradient at A, which is found using calculus. **Can be implied by later working**	A1
	So, $m_N = \dfrac{1}{4}$	Applies $m_N = \dfrac{-1}{m_T}$, to find a numerical m_N, where m_T is found from using calculus. **Can be implied by later working**	M1
	• $y - 10 = \dfrac{1}{4}\left(x - \dfrac{5}{2}\right)$ • $10 = \dfrac{1}{4}\left(\dfrac{5}{2}\right) + c \Rightarrow c = \dfrac{75}{8} \Rightarrow y = \dfrac{1}{4}x + \dfrac{75}{8}$	Correct line method for a **normal** where a numerical $m_N\,(\neq m_T)$ is found from using calculus. **Can be implied by later working**	M1
	leading to $8y - 2x - 75 = 0$ (*)	Correct solution only	A1
			(5)

Question	Scheme		Marks
6(c)	$y=\dfrac{25}{x} \Rightarrow 8\left(\dfrac{25}{x}\right)-2x-75=0$ **or** $x=\dfrac{25}{y} \Rightarrow 8y-2\left(\dfrac{25}{y}\right)-75=0$ **or** $x=5t,\ y=\dfrac{5}{t} \Rightarrow 8(5t)-2\left(\dfrac{5}{t}\right)-75=0$ Substitutes $y=\dfrac{25}{x}$ **or** $x=\dfrac{25}{y}$ **or** $x=5t$ **and** $y=\dfrac{5}{t}$ into the printed equation or their normal equation to obtain an equation in either x only, y only or t only		M1
	$2x^2+75x-200=0$ **or** $8y^2-75y-50=0$ **or** $2t^2+15t-8=0$ **or** $10t^2+75t-40=0$		
	$(2x-5)(x+40)=0 \Rightarrow x=...$ **or** $(y-10)(8y+5)=0 \Rightarrow y=...$ **or** $(2t-1)(t+8)=0 \Rightarrow t=...$ **dependent on the previous M mark** Correct attempt of solving a 3TQ to find either $x=...$, $y=...$ or $t=...$		dM1
	Finds at least one of either $x=-40$ or $y=-\dfrac{5}{8}$		A1
	$B\left(-40,\ -\dfrac{5}{8}\right)$	Both correct coordinates (If coordinates are not stated they can be paired together as $x=...,\ y=...$)	A1
			(4)
			(10 marks)

Notes:

(a) A conclusion is not required on this occasion in part (a).

B1: Condone reference to $c=5$ (as $xy=c^2$ and $\left(ct,\dfrac{c}{t}\right)$ are referred in the Formula book.)

(b)

$\dfrac{dy}{dx}=\dfrac{dy}{dt}.\dfrac{dt}{dx}=-\dfrac{5}{t^2}\left(\dfrac{1}{5}\right)=-\dfrac{1}{t^2} \Rightarrow m_N=t^2 \Rightarrow y-10=t^2\left(x-\dfrac{5}{2}\right)$ **scores only the first M1.**

When $t=\dfrac{1}{2}$ is substituted giving $y-10=\dfrac{1}{4}\left(x-\dfrac{5}{2}\right)$ the response then automatically gets A1(implied) M1(implied) M1

Question 6 notes *continued*

(c)

You can imply the final three marks (dM1A1A1) for either

- $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$
- $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$
- $8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow \left(-40, -\frac{5}{8}\right)$

with no intermediate working.

You can also imply the middle dM1A1 marks for either

- $8\left(\frac{25}{x}\right) - 2x - 75 = 0 \rightarrow x = -40$
- $8y - 2\left(\frac{25}{y}\right) - 75 = 0 \rightarrow y = -\frac{5}{8}$
- $8(5t) - 2\left(\frac{5}{t}\right) - 75 = 0 \rightarrow x = -40$ or $y = -\frac{5}{8}$

with no intermediate working.

Writing $x = -40,\ y = -\frac{5}{8}$ followed by $B\left(40, \frac{5}{8}\right)$ or $B\left(-\frac{5}{8}, -40\right)$ is final A0.

Ignore stating $B\left(\frac{5}{2}, 10\right)$ in addition to $B\left(-40, -\frac{5}{8}\right)$

Question	Scheme		Marks
7(a)	Rotation	Rotation	B1
	67 degrees (anticlockwise)	Either $\arctan\left(\frac{12}{5}\right)$, $\tan^{-1}\left(\frac{12}{5}\right)$, $\sin^{-1}\left(\frac{12}{13}\right)$, $\cos^{-1}\left(\frac{5}{13}\right)$, awrt 67 degrees, awrt 1.2, truncated 1.1 (anticlockwise), awrt 293 degrees clockwise or awrt 5.1 clockwise	B1 o.e.
	about $(0, 0)$	**The mark is dependent on at least one of the previous B marks being awarded.** About $(0, 0)$ or about O or about the origin	dB1
	Note: Give 2nd B0 for 67 degrees clockwise o.e.		**(3)**
(b)	$\{\mathbf{Q} =\}\ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Correct matrix	B1
			(1)
(c)	$\{\mathbf{R} = \mathbf{PQ}\}\ \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; = \begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}$	Multiplies **P** by their **Q** in the correct order and finds at least one element	M1
		Correct matrix	A1
			(2)
(d)	**Way 1**		
	$\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}\begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix}$	Forming the equation "their matrix $\mathbf{R}$" $\begin{pmatrix} x \\ kx \end{pmatrix} = \begin{pmatrix} x \\ kx \end{pmatrix}$ **Allow x being replaced by any non-zero number eg. 1.** Can be implied by at least one correct ft equations below.	M1
	$-\frac{12}{13}x + \frac{5kx}{13} = x$ or $\frac{5}{13}x + \frac{12kx}{13} = kx \Rightarrow k = ...$	Uses their matrix equation to form an equation in k and progresses to give $k =$ numerical value	M1
	So $k = 5$	**dependent on only the previous M mark** $k = 5$	A1 **cao**
	Dependent on all previous marks being scored in this part. Either • Solves **both** $-\frac{12}{13}x + \frac{5kx}{13} = x$ **and** $\frac{5}{13}x + \frac{12kx}{13} = kx$ to give $k = 5$ • Finds $k = 5$ and checks that it is true for the other component • Confirms that $\begin{pmatrix} -\frac{12}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{12}{13} \end{pmatrix}\begin{pmatrix} x \\ 5x \end{pmatrix} = \begin{pmatrix} x \\ 5x \end{pmatrix}$		A1 **cso**
			(4)

Question	Scheme		Marks
7(d) *continued*	**Way 2**		
	Either $\cos 2\theta = -\frac{12}{13},\ \sin 2\theta = \frac{5}{13}$ or $\tan 2\theta = -\frac{5}{12}$	Correct follow through equation in 2θ based on their matrix **R**	M1
		Full method of finding 2θ, then θ and applying $\tan\theta$	M1
	$\{k =\}\ \tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$	$\tan\left(\frac{1}{2}\arccos\left(-\frac{12}{13}\right)\right)$ or $\tan(\text{awrt } 78.7^\circ)$ or $\tan(\text{awrt } 1.37)$. **Can be implied.**	A1
	So $k = 5$	$k = 5$ **by a correct solution only**	A1
			(4)
			(10 marks)

Notes:

(a)

Condone "Turn" for the 1st B1 mark.

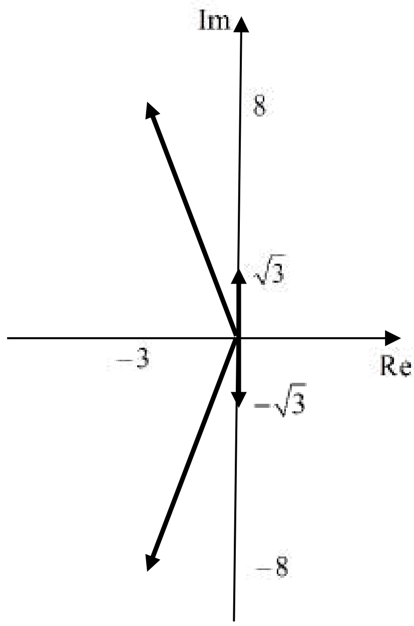
Penalise the first B1 mark for candidates giving a combination of transformations.

(c)

Allow 1st M1 for eg. $\text{"their matrix } \mathbf{R}\text{"}\begin{pmatrix}1\\k\end{pmatrix} = \begin{pmatrix}1\\k\end{pmatrix}$ or $\text{"their matrix } \mathbf{R}\text{"}\begin{pmatrix}k\\k^2\end{pmatrix} = \begin{pmatrix}k\\k^2\end{pmatrix}$

or $\text{"their matrix } \mathbf{R}\text{"}\begin{pmatrix}\frac{1}{k}\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{k}\\1\end{pmatrix}$ or equivalent

$$y = (\tan\theta)x : \begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix} = \begin{pmatrix}-\frac{12}{13} & \frac{5}{13}\\ \frac{5}{13} & \frac{12}{13}\end{pmatrix}$$

Question	Scheme		Marks
8(a)	$f(z) = z^4 + 6z^3 + 76z^2 + az + b$, a, b are real constants. $z_1 = -3 + 8\text{i}$ is given.		
	$-3 - 8\text{i}$	$-3 - 8\text{i}$	B1
			(1)
(b)	$z^2 + 6z + 73$	Attempt to expand $(z-(-3+8\text{i}))(z-(-3-8\text{i}))$ or any valid method ***to establish a quadratic factor*** eg $z = -3 \pm 8\text{i} \Rightarrow z+3 = \pm 8\text{i} \Rightarrow z^2 + 6z + 9 = -64$ or sum of roots -6, product of roots 73 to give $z^2 \pm (\text{sum})z + \text{product}$	M1
		$z^2 + 6z + 73$	A1
	$f(z) = (z^2 + 6z + 73)(z^2 + 3)$	Attempts to find the other quadratic factor. eg. using long division to get as far as $z^2 + ...$ or eg. $f(z) = (z^2 + 6z + 73)(z^2 + ...)$	M1
		$z^2 + 3$	A1
	$\left\{z^2 + 3 = 0 \Rightarrow z = \right\}$ $\pm\sqrt{3}\,\text{i}$	**dependent on only the previous M mark** Correct method of solving the 2nd quadratic factor	dM1
		$\sqrt{3}\,\text{i}$ **and** $-\sqrt{3}\,\text{i}$	A1
			(6)
(c)		<u>**Criteria**</u> • $-3 \pm 8\text{i}$ plotted correctly in quadrants 2 and 3 with some evidence of symmetry • Their other two ***complex roots*** (which are found from solving their 2nd quadratic in (b)) are plotted correctly with some evidence of symmetry about the x-axis	
		Satisfies at least one of the two criteria	B1 ft
		Satisfies both criteria with some indication of scale or coordinates stated. All points (arrows) must be in the correct positions relative to each other.	B1 ft
			(2)
			(9 marks)

Question 8 *continued*
Notes:
(b) Give 3rd M1 for $z^2 + k = 0,\ k > 0 \Rightarrow$ **at least one of either** $z = \sqrt{k}\,\mathrm{i}$ **or** $z = -\sqrt{k}\,\mathrm{i}$ Give 3rd M0 for $z^2 + k = 0,\ k > 0 \Rightarrow z = \pm k\mathrm{i}$ Give 3rd M0 for $z^2 + k = 0,\ k > 0 \Rightarrow z = \pm k$ or $z = \pm\sqrt{k}$ Candidates do not need to find $a = 18,\ b = 219$

Question	Scheme		Marks
9(a)	$2x^2+4x-3=0$ has roots α, β		
	$\alpha+\beta=-\frac{4}{2}$ or -2, $\alpha\beta=-\frac{3}{2}$	**Both** $\alpha+\beta=-\frac{4}{2}$ **and** $\alpha\beta=-\frac{3}{2}$. This may be seen or implied anywhere in this question.	B1
(i)	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=......$	**Use** of a **correct** identity for $\alpha^2+\beta^2$ (May be implied by their work)	M1
	$=(-2)^2-2\left(-\frac{3}{2}\right)=7$	7 **from correct working**	A1 cso
(ii)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=......$ or $=(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)=......$	**Use** of an **appropriate** and **correct** identity for $\alpha^3+\beta^3$ (May be implied by their work)	M1
	$=(-2)^3-3\left(-\frac{3}{2}\right)(-2)=-17$ or $=(-2)\left(7--\frac{3}{2}\right)=-17$	-17 **from correct working**	A1 cso
			(5)
(b)	Sum $=\alpha^2+\beta+\beta^2+\alpha$ $=\alpha^2+\beta^2+\alpha+\beta$ $=7+(-2)=5$	Uses at least one of their $\alpha^2+\beta^2$ or $\alpha+\beta$ in an attempt to find a **numerical value** for the sum of $(\alpha^2+\beta)$ and $(\beta^2+\alpha)$	M1
	Product $=(\alpha^2+\beta)(\beta^2+\alpha)$ $=(\alpha\beta)^2+\alpha^3+\beta^3+\alpha\beta$ $=\left(-\frac{3}{2}\right)^2-17-\frac{3}{2}=-\frac{65}{4}$	Expands $(\alpha^2+\beta)(\beta^2+\alpha)$ and uses at least one of their $\alpha\beta$ or $\alpha^3+\beta^3$ in an attempt to find a **numerical value** for the product of $(\alpha^2+\beta)$ and $(\beta^2+\alpha)$	M1
	$x^2-5x-\frac{65}{4}=0$	Applies $x^2-(\text{sum})x+\text{product}$ (Can be implied) ("$=0$" not required)	M1
	$4x^2-20x-65=0$	Any integer multiple of $4x^2-20x-65=0$, including the "$=0$"	A1
			(4)

Question	Scheme		Marks
9(b) *continued*	**Alternative:** Finding $\alpha^2+\beta$ and $\beta^2+\alpha$ explicitly		
	Eg. Let $\alpha=\dfrac{-4+\sqrt{40}}{4}$, $\beta=\dfrac{-4+\sqrt{40}}{4}$ and so $\alpha^2+\beta=\dfrac{5-3\sqrt{10}}{2}$, $\beta^2+\alpha=\dfrac{5+3\sqrt{10}}{2}$		
	$\left(x-\left(\dfrac{5-3\sqrt{10}}{2}\right)\right)\left(x-\left(\dfrac{5+3\sqrt{10}}{2}\right)\right)$	Uses $\left(x-\left(\alpha^2+\beta\right)\right)\left(x-\left(\beta^2+\alpha\right)\right)$ with exact numerical values. (May expand first)	M1
	$=x^2-\left(\dfrac{5+3\sqrt{10}}{2}\right)x-\left(\dfrac{5-3\sqrt{10}}{2}\right)x+\left(\dfrac{5-3\sqrt{10}}{2}\right)\left(\dfrac{5+3\sqrt{10}}{2}\right)$	Attempts to expand using exact numerical values for $\alpha^2+\beta$ and $\beta^2+\alpha$	M1
	$\Rightarrow x^2-5x-\frac{65}{4}=0$	Collect terms to give a 3TQ. (" = 0" not required)	M1
	$4x^2-20x-65=0$	Any integer multiple of $4x^2-20x-65=0$, including the "= 0"	A1
			(4)
			(9 marks)

Notes:

(a)

1st A1: $\alpha+\beta=2,\ \alpha\beta=-\dfrac{3}{2}\Rightarrow\alpha^2+\beta^2=4-2\left(-\tfrac{3}{2}\right)=7$ is M1A0 cso

Finding $\alpha+\beta=-2, \alpha\beta=-\dfrac{3}{2}$ by writing down or applying $\dfrac{-4+\sqrt{40}}{4}, \dfrac{-4+\sqrt{40}}{4}$ but then writing $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=4+3=7$ and $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=-8-9=-17$ scores B0M1A0M1A0 in part (a).

Applying $\dfrac{-4+\sqrt{40}}{4}, \dfrac{-4+\sqrt{40}}{4}$ explicitly in part (a) will score B0M0A0M0A0

Eg: Give no credit for $\left(\dfrac{-4+\sqrt{40}}{4}\right)^2+\left(\dfrac{-4+\sqrt{40}}{4}\right)^2=7$

or for $\left(\dfrac{-4+\sqrt{40}}{4}\right)^3+\left(\dfrac{-4+\sqrt{40}}{4}\right)^3=-17$

(b)

Candidates **are allowed** to apply $\dfrac{-4+\sqrt{40}}{4}, \dfrac{-4+\sqrt{40}}{4}$ explicitly in part (b).

A correct method leading to a candidate stating $a=4, b=-20, c=-65$ without writing a final answer of $4x^2-20x-65=0$ is **final** M1A0

Question	Scheme		Marks
10	$u_1 = 5,\ u_{n+1} = 3u_n + 2,\ n \geq 1$. Required to prove the result, $u_n = 2\times(3)^n - 1,\ n \in \mathbb{Z}^+$		
(i)	$n=1:\ \ u_1 = 2(3) - 1 = 5$	$u_1 = 2(3) - 1 = 5$ or $u_1 = 6-1 = 5$	B1
	(Assume the result is true for $n = k$)		
	$u_{k+1} = 3\left(2(3)^k - 1\right) + 2$	Substitutes $u_k = 2(3)^k - 1$ into $u_{k+1} = 3u_k + 2$	M1
	$= 2(3)^{k+1} - 1$	**dependent on the previous M mark** Expresses u_{k+1} in term of 3^{k+1}	dM1
		$u_{k+1} = 2(3)^{k+1} - 1$ **by correct solution only**	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n		A1 cso
			(5)
	Required to prove the result $\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3+2n)}{3^n}\ ,\ n \in \mathbb{Z}^+$		
(ii)	$n=1: \text{LHS} = \frac{4}{3}\ , \text{RHS} = 3 - \frac{5}{3} = \frac{4}{3}$	Shows or states **both** $\text{LHS} = \frac{4}{3}$ **and** $\text{RHS} = \frac{4}{3}$ **or** states $\text{LHS} = \text{RHS} = \frac{4}{3}$	B1
	(Assume the result is true for $n = k$)		
	$\sum_{r=1}^{k+1} \frac{4r}{3^r} = 3 - \frac{(3+2k)}{3^k} + \frac{4(k+1)}{3^{k+1}}$	Adds the $(k+1)^{\text{th}}$ term to the sum of k terms	M1
	$= 3 - \frac{3(3+2k)}{3^{k+1}} + \frac{4(k+1)}{3^{k+1}}$	**dependent on the previous M mark** Makes 3^{k+1} or $(3)3^k$ a common denominator for their fractions.	dM1
		Correct expression with common denominator 3^{k+1} or $(3)3^k$ for their fractions.	A1
	$= 3 - \left(\frac{3(3+2k) - 4(k+1)}{3^{k+1}}\right)$ $= 3 - \left(\frac{5+2k}{3^{k+1}}\right)$		
	$= 3 - \frac{(3+2(k+1))}{3^{k+1}}$	$3 - \frac{(3+2(k+1))}{3^{k+1}}$ **by correct solution only**	A1
	If the result is true for $n = k$, then it is true for $n = k+1$. As the result has been shown to be true for $n = 1$, then the result is true for all n		A1 cso
			(6)
			(11 marks)

Question 10 *continued*
Notes:
(i) & (ii) **Final A1 for parts (i) and (ii)** is dependent on all previous marks being scored in that part. It is gained by candidates conveying the ideas of **all** four underlined points **either** at the end of their solution **or** as a narrative in their solution.
(i) $u_1 = 5$ by itself is not sufficient for the 1st B1 mark in part (i). $u_1 = 3 + 2$ without stating $u_1 = 2(3) - 1 = 5$ or $u_1 = 6 - 1 = 5$ is B0
(ii) $\text{LHS} = \text{RHS}$ by itself is not sufficient for the 1st B1 mark in part (ii).

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics FP2

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WFM12/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. Using algebra, find the set of values of x for which

$$\frac{x}{x+2} < \frac{2}{x+5}$$

(7)

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Question 1 continued

Q1

(Total for Question 1 is 7 marks)

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2. (a) Express $\dfrac{1}{(r+6)(r+8)}$ in partial fractions.

(1)

(b) Hence show that

$$\sum_{r=1}^{n} \frac{2}{(r+6)(r+8)} = \frac{n(an+b)}{56(n+7)(n+8)}$$

where a and b are integers to be found.

(4)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total for Question 2 is 5 marks)

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3. (a) Show that the substitution $z = y^{-2}$ transforms the differential equation

$$\frac{dy}{dx} + 2xy = xe^{-x^2}y^3 \quad \text{(I)}$$

into the differential equation

$$\frac{dz}{dx} - 4xz = -2xe^{-x^2} \quad \text{(II)}$$

(4)

(b) Solve differential equation (II) to find z as a function of x.

(5)

(c) Hence find the general solution of differential equation (I), giving your answer in the form $y^2 = f(x)$.

(1)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total for Question 3 is 10 marks)

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4. A transformation T from the z-plane to the w-plane is given by

$$w = \frac{z - 1}{z + 1}, \quad z \neq -1$$

The line in the z-plane with equation $y = 2x$ is mapped by T onto the curve C in the w-plane.

(a) Show that C is a circle and find its centre and radius.

(7)

The region $y < 2x$ in the z-plane is mapped by T onto the region R in the w-plane.

(b) Sketch circle C on an Argand diagram and shade and label region R.

(2)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total for Question 4 is 9 marks)

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5. Given that $y = \cot x$,

(a) show that

$$\frac{d^2y}{dx^2} = 2\cot x + 2\cot^3 x$$

(3)

(b) Hence show that

$$\frac{d^3y}{dx^3} = p\cot^4 x + q\cot^2 x + r$$

where p, q and r are integers to be found.

(3)

(c) Find the Taylor series expansion of $\cot x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$ up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$.

(3)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total for Question 5 is 9 marks)

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6. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2\sin x \quad \text{(I)}$$

(8)

Given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

(b) find the particular solution of differential equation (I).

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

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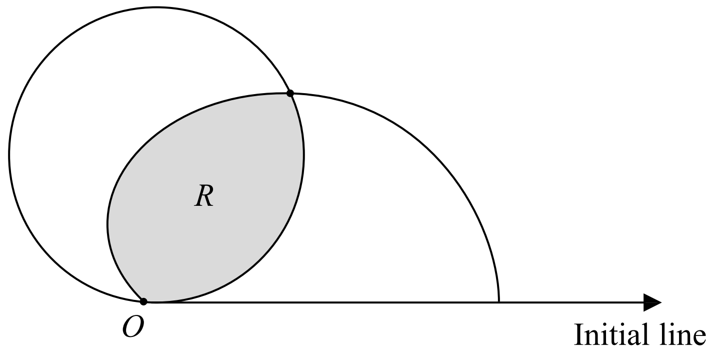

Figure 1

Figure 1 shows the two curves given by the polar equations

$$r = \sqrt{3}\,\sin\theta, \quad 0 \leqslant \theta \leqslant \pi$$

$$r = 1 + \cos\theta, \quad 0 \leqslant \theta \leqslant \pi$$

(a) Verify that the curves intersect at the point P with polar coordinates $\left(\frac{3}{2}, \frac{\pi}{3}\right)$.

(2)

The region R, bounded by the two curves, is shown shaded in Figure 1.

(b) Use calculus to find the exact area of R, giving your answer in the form $a\left(\pi - \sqrt{3}\right)$, where a is a constant to be found.

(6)

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Question 7 continued

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Question 7 continued

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Q7

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8. (a) Show that

$$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where k is a constant to be found.

(3)

Given that $z = \cos\theta + \mathrm{i}\sin\theta$, where θ is real,

(b) show that

(i) $z^n + \dfrac{1}{z^n} = 2\cos n\theta$

(ii) $z^n - \dfrac{1}{z^n} = 2\mathrm{i}\sin n\theta$

(3)

(c) Hence show that

$$\cos^3\theta\,\sin^3\theta = \frac{1}{32}\,(3\sin 2\theta - \sin 6\theta)$$

(4)

(d) Find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3\theta\,\sin^3\theta\,\mathrm{d}\theta$$

(4)

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Question 8 continued

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Question 8 continued

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Q8

(Total for Question 8 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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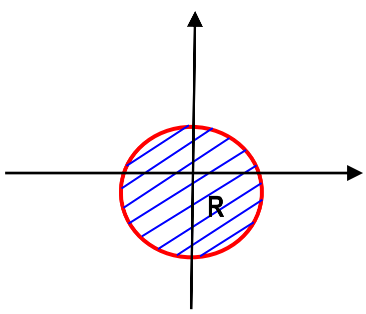
Further Pure Mathematics FP2 Mark scheme

Question	Scheme		Marks
1	$\frac{x}{x+2} < \frac{2}{x+5}$		
	Critical Values -2 and -5	Seen anywhere in solution Both correct B1B1; one correct B1B0	B1 B1
	$\frac{x}{x+2} - \frac{2}{x+5} < 0$		
	$\frac{x^2+3x-4}{(x+2)(x+5)} < 0$		
	$\frac{(x+4)(x-1)}{(x+2)(x+5)} < 0$	Attempt single fraction and factorise numerator or use quad formula	M1
	Critical values -4 and 1	Correct critical values May be seen on a graph or number line.	A1
	$-5 < x < -4,\ -2 < x < 1$ $(-5,-4)\cup(-2,1)$	dM1: Attempt an interval inequality using one of -2 or -5 with another cv A1, A1: Correct intervals Can be in set notation One correct scores A1A0 Award on basis of the inequalities seen - ignore any and/or between them Set notation answers do not need the union sign.	dM1 A1 A1
			(7)
	Alternative		
	Critical Values -2 and -5	Seen anywhere in solution	B1, B1
	$\frac{x}{x+2} < \frac{2}{x+5} \Rightarrow x(x+5)^2(x+2) < 2(x+2)^2(x+5)$		
	$\Rightarrow (x+5)(x+2)\left[x(x+5)-2(x+2)\right] < 0$		
	$\Rightarrow (x+5)(x+2)\left[(x-1)(x+4)\right] < 0$	Multiply by $(x+5)^2(x+2)^2$ and attempt to factorise a quartic or use quad formula	M1
	Critical values -4 and 1	Correct critical values	A1
	$-5 < x < -4,\ -2 < x < 1$ $(-5,-4)\cup(-2,1)$	dM1: Attempt an interval inequality using one of -2 or -5 with another cv A1, A1: Correct intervals Can be in set notation One correct scores A1A0	dM A1 A1
	Any solutions with no algebra (eg sketch graph followed by critical values with no working) scores max B1B1		
			(7 marks)

Question	Scheme		Marks
2(a)	$\dfrac{1}{(r+6)(r+8)}$		
	$\dfrac{1}{2(r+6)}-\dfrac{1}{2(r+8)}$ oe	Correct partial fractions, any equivalent form	B1
			(1)
(b)	$=\left(2\times\dfrac{1}{2}\right)\left(\dfrac{1}{7}-\dfrac{1}{9}+\dfrac{1}{8}-\dfrac{1}{10}+\dfrac{1}{9}-\dfrac{1}{11}...+\dfrac{1}{n+5}-\dfrac{1}{n+7}+\dfrac{1}{n+6}-\dfrac{1}{n+8}\right)$ Expands at least 3 terms at start and 2 at end (may be implied) The partial fractions obtained in (a) can be used without multiplying by 2. Fractions may be $\dfrac{1}{2}\times\dfrac{1}{7}-\dfrac{1}{2}\times\dfrac{1}{9}$ etc These comments apply to both M1 and A1		M1
	$=\dfrac{1}{7}+\dfrac{1}{8}-\dfrac{1}{n+7}-\dfrac{1}{n+8}$	Identifies the terms that do not cancel	A1
	$=\dfrac{15(n+7)(n+8)-56(2n+15)}{56(n+7)(n+8)}$	Attempt common denominator Must have multiplied the fractions from (a) by 2 now	M1
	$=\dfrac{n(15n+113)}{56(n+7)(n+8)}$		A1 cso
			(4)
			(5 marks)

Question	Scheme		Marks
3(a)	$\dfrac{dy}{dx}+2xy=xe^{-x^2}y^3$		
	$z=y^{-2}\Rightarrow y=z^{-\frac{1}{2}}$		
	$\dfrac{dy}{dx}=-\frac{1}{2}z^{-\frac{3}{2}}\dfrac{dz}{dx}$	M1: $\dfrac{dy}{dx}=kz^{-\frac{3}{2}}\dfrac{dz}{dx}$ A1: Correct differentiation	M1 A1
	$-\frac{1}{2}z^{-\frac{3}{2}}\dfrac{dz}{dx}+\dfrac{2x}{z^{\frac{1}{2}}}=xe^{-x^2}z^{-\frac{3}{2}}$	Substitutes for dy/dx	M1
	$\dfrac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1 cso
			(4)
	Alternative 1		
	$\dfrac{dz}{dy}=-2y^{-3}$ oe	M1: $\dfrac{dz}{dy}=ky^{-3}$ A1: Correct differentiation	M1 A1
	$-\frac{1}{2}y^3\dfrac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\dfrac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
	Alternative 2		
	$\dfrac{dz}{dx}=-2y^{-3}\dfrac{dy}{dx}$	M1: $\dfrac{dz}{dx}=ky^{-3}\dfrac{dy}{dx}$ inc chain rule A1: Correct differentiation	M1 A1
	$-\frac{1}{2}y^3\dfrac{dz}{dx}+2xy=xe^{-x^2}y^3$	Substitutes for dy/dx	M1
	$\dfrac{dz}{dx}-4xz=-2xe^{-x^2}$ *	Correct completion to printed answer with no errors seen	A1
(b)	$I=e^{\int -4x\,dx}=e^{-2x^2}$	M1: $I=e^{\int \pm 4x\,dx}$ A1: e^{-2x^2}	M1 A1
	$ze^{-2x^2}=\int -2xe^{-3x^2}\,dx$	$z\times I=\int -2xe^{-x^2}I\,dx$	dM1
	$\dfrac{1}{3}e^{-3x^2}\,(+c)$	$\int xe^{qx^2}\,dx=pe^{qx^2}\,(+c)$	M1
	$z=ce^{2x^2}+\dfrac{1}{3}e^{-x^2}$	Or equivalent	A1
			(5)

Question	Scheme		Marks
3(c)	$\frac{1}{y^2} = ce^{2x^2} + \frac{1}{3}e^{-x^2} \Rightarrow y^2 = \frac{1}{ce^{2x^2} + \frac{1}{3}e^{-x^2}}$	$y^2 = \frac{1}{(b)}\left(= \frac{3e^{x^2}}{1+ke^{3x^2}}\right)$	B1ft
			(1)
			(10 marks)

Question	Scheme		Marks
4(a)	$w=\dfrac{z-1}{z+1}$		
	$w=\dfrac{z-1}{z+1}\Rightarrow wz+w=z-1\Rightarrow z=...$	Attempt to make z the subject	M1
	$z=\dfrac{w+1}{1-w}$	Correct expression in terms of w	A1
	$=\dfrac{u+iv+1}{1-u-iv}\times\dfrac{1-u+iv}{1-u+iv}$	Introduces "u + iv" and multiplies top and bottom by the complex conjugate of the bottom	M1
	$x=\dfrac{-u^2-v^2+1}{....},\quad y=\dfrac{2v}{....}$		
	$y=2x\Rightarrow 2v=-2u^2-2v^2+2$	Uses real and imaginary parts and $y = 2x$ to obtain an equation connecting "u" and "v" Can have the 2 on the wrong side.	M1
	$u^2+\left(v+\frac{1}{2}\right)^2-\frac{1}{4}=1$	Processes their equation to a form that is recognisable as a circle ie coefficients of u^2 and v^2 are the same and no uv terms	M1
	Centre (0, $-\frac{1}{2}$), radius $\dfrac{\sqrt{5}}{2}$	A1: Correct centre (allow -½i) A1: Correct radius	A1,A1
			(7)
	Special Case:		
	$w=\dfrac{x+\mathrm{i}y-1}{x+\mathrm{i}y+1}=\dfrac{(x-1)+2x\mathrm{i}}{(x+1)+2x\mathrm{i}}\times\dfrac{(x+1)-2x\mathrm{i}}{(x+1)-2x\mathrm{i}}$	M1: rationalise the denominator, may have $2x$ or y	
	$=\dfrac{(x^2-1)+4x^2+2x\mathrm{i}(x+1-(x-1))}{(x+1)^2+4x^2}$	A1: Correct result in terms of x only. Must have rational denominator shown, but no other simplification needed	
(b)	 	B1ft: Their circle correctly positioned provided their equation does give a circle B1: Completely correct sketch and shading	B1ft B1
			(2)
			(9 marks)

Question	Scheme		Marks
5(a)	$y = \cot x$		
	$\frac{dy}{dx} = -\mathrm{cosec}^2 x$		
	$\frac{d^2y}{dx^2} = (-2\mathrm{cosec}\, x)(-\mathrm{cosec}\, x \cot x)$	M1: Differentiates using the chain rule or product/quotient rule A1: Correct derivative	M1A1
	$= 2\mathrm{cosec}^2 x \cot x = 2\cot x + 2\cot^3 x$*	A1: Correct completion to printed answer $1+\cot^2 x = \mathrm{cosec}^2 x$ or $\cos^2 x + \sin^2 x = 1$ must be used Full working must be shown	A1cso*
			(3)
	Alternative		
	$y = \frac{\cos x}{\sin x} \to \frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$		
	$\frac{d^2y}{dx^2} = -(-2\sin^{-3} x \cos x) = ...$		M1A1
	Correct completion to printed answer see above		A1
			(3)
(b)	$\frac{d^3y}{dx^3} = -2\mathrm{cosec}^2 x - 6\cot^2 x\, \mathrm{cosec}^2 x$	Correct third derivative	B1
	$= -2(1+\cot^2 x) - 6\cot^2 x(1+\cot^2 x)$	Uses $1+\cot^2 x = \mathrm{cosec}^2 x$	M1
	$= -6\cot^4 x - 8\cot^2 x - 2$	cso	A1
			(3)
(c)	$f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}, f'\left(\frac{\pi}{3}\right) = -\frac{4}{3}, f''\left(\frac{\pi}{3}\right) = \frac{8}{3\sqrt{3}}, f'''\left(\frac{\pi}{3}\right) = -\frac{16}{3}$ M1: Attempts all 4 values at $\frac{\pi}{3}$ No working need be shown		M1
	$(y=)\frac{1}{\sqrt{3}} - \frac{4}{3}\left(x-\frac{\pi}{3}\right) + \frac{4}{3\sqrt{3}}\left(x-\frac{\pi}{3}\right)^2 - \frac{8}{9}\left(x-\frac{\pi}{3}\right)^3$ M1: Correct application of Taylor using their values. Must be up to and including $\left(x-\frac{\pi}{3}\right)^3$ A1: Correct expression Must start $y = ...$ or $\cot x$ f(x) allowed provided defined here or above as $f(x) = \cot x$ or y Decimal equivalents allowed (min 3 sf apart from 0.77), 0.578, 1.33, 0.770, (0.7698.., so accept 0.77) 0.889		M1A1
			(3)
			(9 marks)

Question	Scheme		Marks
6(a)	$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\frac{\mathrm{d}y}{\mathrm{d}x}-3y=2\sin x$$		
	AE: $m^2-2m-3=0$		
	$m^2-2m-3=0\Rightarrow m=...(-1,3)$	Forms Auxiliary Equation and attempts to solve (usual rules)	M1
	$(y=)Ae^{3x}+Be^{-x}$	Cao	A1
	PI: $(y=)p\sin x+q\cos x$	Correct form for PI	B1
	$(y'=)p\cos x-q\sin x$ $(y''=)-p\sin x-q\cos x$		
	$-p\sin x-q\cos x-2(p\cos x-q\sin x)-3p\sin x-3q\cos x=2\sin x$ Differentiates twice and substitutes		M1
	$2q-4p=2,\ 4q+2p=0$	Correct equations	A1
	$p=-\frac{2}{5},\ q=\frac{1}{5}$	A1A1 both correct A1A0 one correct	A1 A1
	$y=\frac{1}{5}\cos x-\frac{2}{5}\sin x$		
	$y=Ae^{3x}+Be^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Follow through their p and q and their CF	B1ft
			(8)
(b)	$y'=3Ae^{3x}-Be^{-x}-\frac{1}{5}\sin x-\frac{2}{5}\cos x$	Differentiates their GS	M1
	$0=A+B+\frac{1}{5},\ 1=3A-B-\frac{2}{5}$	M1: Uses the given conditions to give two equations in A and B A1: Correct equations	M1 A1
	$A=\frac{3}{10},\ B=-\frac{1}{2}$	Solves for A and B Both correct	A1
	$y=\frac{3}{10}e^{3x}-\frac{1}{2}e^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Sub their values of A and B in their GS	A1ft
			(5)
			(13 marks)

Question	Scheme		Marks
7(a)	$\theta=\frac{\pi}{3}\Rightarrow r=\sqrt{3}\sin\left(\frac{\pi}{3}\right)=\frac{3}{2}$	Attempt to verify coordinates in at least one of the polar equations	M1
	$\theta=\frac{\pi}{3}\Rightarrow r=1+\cos\left(\frac{\pi}{3}\right)=\frac{3}{2}$	Coordinates verified in both curves (Coordinate brackets not needed)	A1
			(2)
	Alternative		
	Equate rs: $\sqrt{3}\sin\theta=1+\cos\theta$ and verify (by substitution) that $\theta=\frac{\pi}{3}$ is a solution **or** solve by using $t=\tan\frac{\theta}{2}$ **or** writing $\frac{\sqrt{3}}{2}\sin\theta-\frac{1}{2}\cos\theta=\frac{1}{2}$ $\quad\sin\left(\theta-\frac{\pi}{6}\right)=\frac{1}{2}$ $\quad\theta=\frac{\pi}{3}$ Squaring the original equation allowed as θ is known to be between 0 and π		M1
	Use $\theta=\frac{\pi}{3}$ in either equation to obtain $r=\frac{3}{2}$		A1
			(2)
(b)	$\frac{1}{2}\int(\sqrt{3}\sin\theta)^2\,\mathrm{d}\theta,\quad\frac{1}{2}\int(1+\cos\theta)^2\,\mathrm{d}\theta$	Correct formula used on at least one curve (1/2 may appear later) Integrals may be separate or added or subtracted.	M1
	$=\frac{1}{2}\int 3\sin^2\theta\,\mathrm{d}\theta,\quad\frac{1}{2}\int(1+2\cos\theta+\cos^2\theta)\,\mathrm{d}\theta$		
	$=\left(\frac{1}{2}\right)\int\frac{3}{2}(1-\cos2\theta)\,\mathrm{d}\theta,\quad\left(\frac{1}{2}\right)\int\left(1+2\cos\theta+\frac{1}{2}(1+\cos2\theta)\right)\mathrm{d}\theta$ Attempt to use $\sin^2\theta$ or $\cos^2\theta=\pm\frac{1}{2}\pm\frac{1}{2}\cos2\theta$ on either integral **Not** dependent 1/2 may be missing		M1
	$=\frac{3}{4}\left[\theta-\frac{1}{2}\sin2\theta\right]_{(0)}^{\left(\frac{\pi}{3}\right)},\quad\frac{1}{2}\left[\frac{3}{2}\theta+2\sin\theta+\frac{1}{4}\sin2\theta\right]_{\left(\frac{\pi}{3}\right)}^{(\pi)}$ Correct integration (ignore limits) A1A1 or A1A0		A1, A1
	$R=\frac{3}{4}\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}(-0)\right]+\frac{1}{2}\left[\frac{3\pi}{2}-\left(\frac{\pi}{2}+\sqrt{3}+\frac{\sqrt{3}}{8}\right)\right]$	Correct use of limits for both integrals Integrals must be added. Dep on both previous M marks	**dd**M1
	$=\frac{3}{4}\left(\pi-\sqrt{3}\right)$	Cao No equivalents allowed	A1
			(6)
			(8 marks)

Question	Scheme		Marks
8(a)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^2-\frac{1}{z^2}\right)^3$		
	$=z^6-3z^2+\frac{3}{z^2}-z^{-6}$	M1: Attempt to expand A1: Correct expansion	M1A1
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors seen	A1
			(3)
	Alternative		
	$\left(z+\frac{1}{z}\right)^3=z^3+3z+\frac{3}{z}+\frac{1}{z^3},\ \left(z-\frac{1}{z}\right)^3=z^3-3z+\frac{3}{z}-\frac{1}{z^3}$		M1A1
	M1: Attempt to expand both cubic brackets A1: Correct expansions		
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors	A1
			(3)
(b)(i)(ii)	$z^n=\cos n\theta+i\sin n\theta$	Correct application of de Moivre	B1
	$z^{-n}=\cos(-n\theta)+i\sin(-n\theta)=\pm\cos n\theta\pm\sin n\theta$ but must be different from their z^n	Attempt z^{-n}	M1
	$z^n+\frac{1}{z^n}=2\cos n\theta$*, $z^n-\frac{1}{z^n}=2i\sin n\theta$*	$z^{-n}=\cos n\theta-i\sin n\theta$ must be seen	A1*
			(3)
(c)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=(2\cos\theta)^3(2i\sin\theta)^3$		B1
	$z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)=2i\sin 6\theta-6i\sin 2\theta$	Follow through their k in place of 3	B1ft
	$-64i\sin^3\theta\cos^3\theta=2i\sin 6\theta-6i\sin 2\theta$	Equating right hand sides and simplifying $2^3\times(2\mathrm{i})^3$ (B mark needed for each side to gain M mark)	M1
	$\cos^3\theta\sin^3\theta=\frac{1}{32}(3\sin 2\theta-\sin 6\theta)$ *		A1cso
			(4)

Question	Scheme		Marks
8(d)	$\int_0^{\frac{\pi}{8}} \cos^3\theta\sin^3\theta\,d\theta = \int_0^{\frac{\pi}{8}} \frac{1}{32}\left(3\sin 2\theta - \sin 6\theta\right)d\theta$		
	$= \frac{1}{32}\left[-\frac{3}{2}\cos 2\theta + \frac{1}{6}\cos 6\theta\right]_0^{\frac{\pi}{8}}$	M1: $p\cos 2\theta + q\cos 6\theta$ A1: Correct integration Differentiation scores M0A0	M1 A1
	$= \frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}} - \frac{1}{6\sqrt{2}}\right) - \left(-\frac{3}{2} + \frac{1}{6}\right)\right] = \frac{1}{32}\left(\frac{4}{3} - \frac{5\sqrt{2}}{6}\right)$	dM1: Correct use of limits – lower limit to have non-zero result. Dep on previous M mark A1: Cao (oe) but must be exact	dM1 A1
			(4)
			(14 marks)

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics FP3

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WFM13/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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S 5 9 7 6 0 A 0 1 3 2

Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. The curve C has equation

$$y = 9\cosh x + 3\sinh x + 7x$$

Use differentiation to find the exact x coordinate of the stationary point of C, giving your answer as a natural logarithm.

(6)

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Question 1 continued

Q1

(Total for Question 1 is 6 marks)

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2. An ellipse has equation

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

The point P lies on the ellipse and has coordinates $(5\cos\theta, 2\sin\theta)$, $0 < \theta < \frac{\pi}{2}$

The line L is a normal to the ellipse at the point P.

(a) Show that an equation for L is

$$5x\sin\theta - 2y\cos\theta = 21\sin\theta\cos\theta$$

(5)

Given that the line L crosses the y-axis at the point Q and that M is the midpoint of PQ,

(b) find the exact area of triangle OPM, where O is the origin, giving your answer as a multiple of $\sin 2\theta$

(6)

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Question 2 continued

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Question 2 continued

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Question 2 continued

Q2

(Total for Question 2 is 11 marks)

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3. Without using a calculator, find

(a) $\displaystyle\int_{-2}^{1} \frac{1}{x^2 + 4x + 13}\,\mathrm{d}x$, giving your answer as a multiple of π,

(5)

(b) $\displaystyle\int_{-1}^{4} \frac{1}{\sqrt{4x^2 - 12x + 34}}\,\mathrm{d}x$, giving your answer in the form $p\ln\left(q + r\sqrt{2}\right)$,

where p, q and r are rational numbers to be found.

(7)

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Question 3 continued

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Q3

(Total for Question 3 is 12 marks)

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4.

$$\mathbf{M} = \begin{pmatrix} 1 & k & 0 \\ -1 & 1 & 1 \\ 1 & k & 3 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) Find $\mathbf{M}^{-1}$ in terms of k.

(5)

Hence, given that $k = 0$

(b) find the matrix $\mathbf{N}$ such that

$$\mathbf{MN} = \begin{pmatrix} 3 & 5 & 6 \\ 4 & -1 & 1 \\ 3 & 2 & -3 \end{pmatrix}$$

(4)

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Q4

(Total for Question 4 is 9 marks)

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5. Given that $y = \text{artanh}(\cos x)$

(a) show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\text{cosec } x$$

(2)

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{6}} \cos x \text{ artanh}(\cos x) \, \mathrm{d}x$$

giving your answer in the form $a \ln\left(b + c\sqrt{3}\right) + d\pi$, where a, b, c and d are rational numbers to be found.

(5)

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Question 5 continued

Q5

(Total for Question 5 is 7 marks)

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6. The coordinates of the points A, B and C relative to a fixed origin O are (1, 2, 3), (–1, 3, 4) and (2, 1, 6) respectively. The plane Π contains the points A, B and C.

(a) Find a cartesian equation of the plane Π.

(5)

The point D has coordinates $(k, 4, 14)$ where k is a positive constant.

Given that the volume of the tetrahedron $ABCD$ is 6 cubic units,

(b) find the value of k.

(4)

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Question 6 continued

Question 6 continued

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Question 6 continued

Q6

(Total for Question 6 is 9 marks)

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7. The curve C has parametric equations

$$x = 3t^4, \quad y = 4t^3, \qquad 0 \leqslant t \leqslant 1$$

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is S.

(a) Show that

$$S = k\pi\int_0^1 t^5(t^2+1)^{\frac{1}{2}}\,\mathrm{d}t$$

where k is a constant to be found.

(4)

(b) Use the substitution $u^2 = t^2 + 1$ to find the value of S, giving your answer in the form $p\pi\left(11\sqrt{2} - 4\right)$ where p is a rational number to be found.

(7)

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Question 7 continued

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Question 7 continued

Q7

(Total for Question 7 is 11 marks)

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8. $$I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geqslant 0$$

(a) Show that, for $n \geqslant 1$

$$I_n = I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$$

(5)

(b) Hence show that

$$\int_0^{\ln 2} \tanh^4 x \, dx = p + \ln 2$$

where p is a rational number to be found.

(5)

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Question 8 continued

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Question 8 continued

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Q8

(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

Further Pure Mathematics FP3 Mark scheme

Question	Scheme		Marks
1	$y = 9\cosh x + 3\sinh x + 7x$		
	$\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7$	Correct derivative	B1
	$9\frac{(e^x - e^{-x})}{2} + 3\frac{(e^x + e^{-x})}{2} + 7 = 0$	Replaces sinh$x$ and cosh$x$ by the correct exponential forms	M1
	Note that the first 2 marks can score the other way round: M1: $y = 9\frac{(e^x + e^{-x})}{2} + 3\frac{(e^x - e^{-x})}{2} + 7x$ B1: $\frac{dy}{dx} = 9\frac{(e^x - e^{-x})}{2} + 3\frac{(e^x + e^{-x})}{2} + 7$		
	$12e^{2x} + 14e^x - 6 = 0$ oe	M1: Obtains a quadratic in e^x A1: Correct quadratic	M1 A1
	$(3e^x - 1)(2e^x + 3) = 0 \Rightarrow e^x = ...$	Solves their quadratic as far as $e^x = ...$	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow –ln3) $e^x = -\frac{3}{2}$ need not be seen. Extra answers, award A0	A1
	Alternative		
	$\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7$	Correct derivative	B1
	$9\sinh x = -3\cosh x - 7 \Rightarrow 81\sinh^2 x = 9\cosh^2 x + 42\cosh x + 49$		
	$72\cosh^2 x - 42\cosh x - 130 = 0$	Squares and attempts quadratic in cosh$x$	M1
	$(3\cosh x - 5)(12\cosh x + 13) = 0 \Rightarrow \cosh x = \frac{5}{3}$	M1: Solves quadratic A1: Correct value	M1 A1
	$x = \ln\left(\frac{5}{3} \pm \sqrt{\left(\frac{5}{3}\right)^2 - 1}\right)$	Use of ln form of arcosh	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow – ln3)	A1
	NB: Ignore any attempts to find the y coordinate		
			(6 marks)

Question	Scheme		Marks
2(a)	$\frac{x^2}{25}+\frac{y^2}{4}=1, \quad P(5\cos\theta, 2\sin\theta)$		
	$\frac{\mathrm{d}x}{\mathrm{d}\theta}=-5\sin\theta, \ \frac{\mathrm{d}y}{\mathrm{d}\theta}=2\cos\theta$ or $\frac{2x}{25}+\frac{2y}{4}\frac{\mathrm{d}y}{\mathrm{d}x}=0$	Correct derivatives or correct implicit differentiation	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cos\theta}{-5\sin\theta}$	Divides their derivatives correctly or substitutes and rearranges	M1
	$M_N=\frac{5\sin\theta}{2\cos\theta}$	Correct perpendicular gradient rule	M1
	$y-2\sin\theta=\frac{5\sin\theta}{2\cos\theta}\left(x-5\cos\theta\right)$	Correct straight line method (any complete method) **Must** use their gradient of the normal.	M1
	$5x\sin\theta-2y\cos\theta=21\sin\theta\cos\theta$*	cso	A1*
			(5)
(b)	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	M is $\left(\frac{0+5\cos\theta}{2}, \frac{2\sin\theta-\frac{21}{2}\sin\theta}{2}\right)$ $\left(=\left(\frac{5}{2}\cos\theta, -\frac{17}{4}\sin\theta\right)\right)$	Correct mid-point method for at least one coordinate Can be implied by a correct x coordinate	M1
	L cuts x-axis at $\frac{21}{5}\cos\theta$		B1
	Area $OPM = OLP + OLM$ $\frac{1}{2}.\frac{21}{5}\cos\theta.2\sin\theta+\frac{1}{2}.\frac{21}{5}\cos\theta.\frac{17}{4}\sin\theta$	M1: Correct triangle area method using their coordinates A1: Correct expression	M1 A1
	$=\frac{105}{16}\sin 2\theta$	Or $6.5625\sin 2\theta$ must be positive	A1
			(6)

Question	Scheme		Marks
2(b) *continued*	**Alternative 1: Using Area *OPM***		
	See above for B1M1		B1 M1
	Area $\Delta OPM = \frac{1}{2}\begin{vmatrix} 0 & 5\cos\theta & \frac{5}{2}\cos\theta & 0 \\ 0 & 2\sin\theta & -\frac{17}{4}\sin\theta & 0 \end{vmatrix}$	M1: Correct determinant with their coords, with 2 or 3 points. $\begin{matrix}0\\0\end{matrix}$ should be at both or neither end. A1: Correct determinant (There are more complicated determinants using the 3 points.)	M1 A1
	$=\frac{1}{2}\left(0+5\sin\theta\cos\theta+0-0+\frac{85}{4}\sin\theta\cos\theta-0\right)$	A1	A1
	$=\frac{105}{4}\sin\theta\cos\theta$		
	$=\frac{105}{16}\sin 2\theta$		A1
			(6)
	Alternative 2: Using Area *OPQ*		
	At Q, $x=0 \Rightarrow y=-\frac{21}{2}\sin\theta$		B1
	Area $\Delta OPQ = \frac{1}{2}\begin{vmatrix} 5\cos\theta & 0 \\ 2\sin\theta & -\frac{21}{2}\sin\theta \end{vmatrix}$	Can be implied by the following line	M1 A1
	$=\frac{1}{2}\times\frac{105}{2}\sin\theta\cos\theta$	OQ is base, x coord of P is height	A1
	$=\frac{105}{8}\sin 2\theta$		
	Area $OPM = \frac{1}{2}$ Area OPQ		M1
	$=\frac{105}{16}\sin 2\theta$		A1
			(6)

Question	Scheme		Marks
2(b) *continued*	**Alternative 3**		
	At Q, $x = 0 \Rightarrow y = -\frac{21}{2}\sin\theta$		B1
	M is $\left(\frac{0+5\cos\theta}{2}, \frac{2\sin\theta - \frac{21}{2}\sin\theta}{2}\right)$ $\quad\left(=\left(\frac{5}{2}\cos\theta, -\frac{17}{4}\sin\theta\right)\right)$		M1
	$OP = \sqrt{4\sin^2\theta + 25\cos^2\theta}$ $\left(=\sqrt{4+21\cos^2\theta}\right)$		B1
	$d = \dfrac{\frac{5}{2}\cos\theta \times \frac{2\sin\theta}{5\cos\theta} + \frac{17}{4}\sin\theta}{\sqrt{\frac{4\sin^2\theta}{25\cos^2\theta}+1}} = \dfrac{\frac{21}{4}\sin\theta}{\sqrt{\frac{4+21\cos^2\theta}{25\cos^2\theta}}}$		
	Area $= \frac{1}{2} \times \dfrac{\frac{21}{4}\sin\theta}{\sqrt{\frac{4+21\cos^2\theta}{25\cos^2\theta}}} \times \sqrt{4+21\cos^2\theta}$		M1 A1
	$= \frac{105}{16}\sin 2\theta$		A1
			(6)
			(11 marks)

Question	Scheme		Marks
3(a)	$x^2+4x+13\equiv(x+2)^2+9$		B1
	$\int\frac{1}{(x+2)^2+9}\,dx=\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)$	M1: $k\arctan f(x)$. A1: Correct expression	M1 A1
	$\left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{1}=\frac{1}{3}(\arctan 1-\arctan 0)$	Correct use of limits arctan0 need not be shown	M1
	$\frac{\pi}{12}$	cao	A1
			(5)
	Alternative		
	Sub $x+2=3\tan t$		
	$x^2+4x+13\equiv(x+2)^2+9$		B1
	$\frac{dx}{dt}=3\sec^2 t\qquad x=-2,\ \tan t=0,\ t=0;\ x=1,\tan t=1,\ t=\frac{\pi}{4}$		
	$\int\frac{3\sec^2 t}{9\tan^2 t+9}\,dt=\frac{1}{3}\int dt=\frac{1}{3}t$	M1 sub and integrate inc use of $\tan^2+1=\sec^2$ A1 Correct expression Ignore limits	M1 A1
	$\left[\frac{\pi}{12}\right]_0^{\frac{\pi}{4}}$.	Either change limits and substitute Or reverse substitution and substitute original imits	M1
	$\frac{\pi}{12}$	cao	A1
			(5)

Question	Scheme		Marks
3(b)	$4x^2-12x+34=4\left(x-\frac{3}{2}\right)^2+25$ or $(2x-3)^2+25$	M1: $4(x\pm p)^2\pm q,\ (p,q\neq 0)$ A1: $4\left(x-\frac{3}{2}\right)^2+25$	M1 A1
	$\int\frac{1}{\sqrt{4\left(x-\frac{3}{2}\right)^2+25}}\,\mathrm{d}x=\frac{1}{2}\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2+\frac{25}{4}}}\,\mathrm{d}x=\frac{1}{2}\text{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)$ M1: $k\text{arsinh f}(x)$. A1: Correct expression		M1 A1
	$\left[\frac{1}{2}\text{arsinh}\left(\frac{x-\frac{3}{2}}{\frac{5}{2}}\right)\right]_{-1}^{4}=\frac{1}{2}(\text{ar}\sinh(1)-\text{ar}\sinh(-1))$	Correct use of limits	M1
	$=\frac{1}{2}\left(\ln\left(1+\sqrt{2}\right)-\ln\left(-1+\sqrt{2}\right)\right)$	Uses the logarithmic form of arsinh	M1
	$=\frac{1}{2}\ln\left(3+2\sqrt{2}\right)$ or $\ln\left(1+\sqrt{2}\right)$	cao	A1
			(7)
	Alternative: Second M1 A1		
	Sub $2x-3=u$ or $2x-3=5\sinh u$		
	$\int_{\text{arsinh}-1}^{\text{arsinh}1}\frac{1}{\sqrt{25\sinh^2u+25}}5\cosh u\,\mathrm{d}u=\left[\frac{1}{2}\text{arsinh}\left(\frac{u}{5}\right)\right]_{-5}^{5}$		M1 A1
	$\int_{-5}^{5}\frac{1}{2\sqrt{u^2+25}}\,\mathrm{d}u=\left[\frac{1}{2}\text{arsinh}\left(\frac{u}{5}\right)\right]_{-5}^{5}$		
			(12 marks)

Question	Scheme		Marks
4(a)	$\mathbf{M}=\begin{pmatrix}1 & k & 0\\ -1 & 1 & 1\\ 1 & k & 3\end{pmatrix}$		
	$\lvert\mathbf{M}\rvert=3-k-k(-3-1)(=3k+3)$	Correct determinant in any form	B1
	$\mathbf{M}^{\mathrm{T}}=\begin{pmatrix}1 & -1 & 1\\ k & 1 & k\\ 0 & 1 & 3\end{pmatrix}$ or minors $\begin{pmatrix}3-k & -4 & -k-1\\ 3k & 3 & 0\\ k & 1 & 1+k\end{pmatrix}$ or cofactors $\begin{pmatrix}3-k & 4 & -k-1\\ -3k & 3 & 0\\ k & -1 & 1+k\end{pmatrix}$		B1
	$\mathbf{M}^{-1}=\dfrac{1}{3+3k}\begin{pmatrix}3-k & -3k & k\\ 4 & 3 & -1\\ -k-1 & 0 & 1+k\end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1ft: Two rows or two columns correct (**follow through their determinant** but not incorrect entries in the matrices used) A1ft: Fully correct inverse (follow through as before)	M1 A1ft A1ft
	NB: If every element is the negative of the correct element, allow M1A1A0		
			(5)
(b)	$\mathbf{MN}=\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}\Rightarrow\mathbf{N}=\mathbf{M}^{-1}\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}$	Correct statement	B1
	$\mathbf{N}=\dfrac{1}{3}\begin{pmatrix}3 & 0 & 0\\ 4 & 3 & -1\\ -1 & 0 & 1\end{pmatrix}\begin{pmatrix}3 & 5 & 6\\ 4 & -1 & 1\\ 3 & 2 & -3\end{pmatrix}=\begin{pmatrix}3 & 5 & 6\\ 7 & 5 & 10\\ 0 & -1 & -3\end{pmatrix}$	M1: Multiplies the given matrix by their $\mathbf{M}^{-1}$ in the correct order Must include the "$\frac{1}{3}$" A2: Correct matrix (-1 each error). If left with $\frac{1}{3}$ outside the matrix award A0	M1 A(2, 1, 0)
			(4)
			(9 marks)

Question	Scheme		Marks
5(a)	$y = \text{artanh}(\cos x)$		
	$\frac{dy}{dx} = \frac{1}{1-\cos^2 x} \times -\sin x$	Correct use of the chain rule	M1
	$= \frac{-\sin x}{\sin^2 x} = \frac{-1}{\sin x} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
			(2)
	Alternative 1		
	$\tanh y = \cos x \Rightarrow \text{sech}^2 y \frac{dy}{dx} = -\sin x$		
	$\frac{dy}{dx} = \frac{-\sin x}{\text{sech}^2 y} = \frac{-\sin x}{1-\cos^2 x}$	Correct differentiation to obtain a function of x	M1
	$= \frac{-\sin x}{\sin^2 x} = \frac{-1}{\sin x} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
			(2)
	Alternative 2		
	$\text{artanh}(\cos x) = \frac{1}{2}\ln\left(\frac{1+\cos x}{1-\cos x}\right)$		
	$\frac{dy}{dx} = \frac{1}{2} \times \frac{1-\cos x}{1+\cos x} \times \frac{-\sin x(1-\cos x) - \sin x(1+\cos x)}{(1-\cos x)^2}$	Correct differentiation to obtain a function of x	M1
	$= \frac{-2\sin x}{2(1-\cos^2 x)} = -\text{cosec}\, x$ *	A1: Correct completion with no errors	A1
			(2)
(b)	$\int \cos x \,\text{artanh}(\cos x)\,dx = \sin x \,\text{artanh}(\cos x) - \int \sin x \times -\text{cosec} x\,dx$ M1: Parts in the correct direction A1: Correct expression		M1 A1
	$\left[\sin x \,\text{artanh}(\cos x) + x\right]_0^{\frac{\pi}{6}} = \frac{1}{2}\text{artanh}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}(-(0))$ M1: Correct use of limits on either part (provided both parts are integrated). Lower limit need not be shown		M1
	$= \frac{1}{4}\ln\left(\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}\right) + \frac{\pi}{6}$	Use of the logarithmic form of artanh	M1
	$= \frac{1}{4}\ln(7+4\sqrt{3}) + \frac{\pi}{6}$ or $\frac{1}{2}\ln(2+\sqrt{3}) + \frac{\pi}{6}$	Cao (oe)	A1
	The last 2 M marks may be gained in reverse order.		**(5)**
			(7 marks)

Question	Scheme		Marks
6(a)	$\overrightarrow{AB}=\begin{pmatrix}-2\\1\\1\end{pmatrix},\ \overrightarrow{AC}=\begin{pmatrix}1\\-1\\3\end{pmatrix},\ \overrightarrow{BC}=\begin{pmatrix}3\\-2\\2\end{pmatrix}$	Two correct vectors in Π Can be negatives of those shown	B1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 1 & 1\\1 & -1 & 3\end{vmatrix}=\begin{pmatrix}4\\7\\1\end{pmatrix}$	M1: Attempt cross product of two vectors lying in Π (At least one no. to be correct.) A1: Correct normal vector	M1 A1
	$\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\3\end{pmatrix}=4+14+3$	Attempt scalar product with their normal and a point in the plane	dM1
	$4x+7y+z=21$	Cao (oe)	A1
			(5)
	Alternative 1		
	$a+2b+3c=d$ $-a+3b+4c=d$ $2a+b+6c=d$	Correct equations	B1
	$a=\frac{4}{21}d,\ b=\frac{1}{3}d,\ c=\frac{1}{21}d$	M1: Solve for a, b and c in terms of d A1: Correct equations	M1 A1
	$d=21\Rightarrow a=...,\ b=...,\ c=...$	Obtains values for a, b, c and d	M1
	$4x+7y+z=21$	Cao (oe)	A1
			(5)
	Alternative 2: Using $\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$ where **b** and **c** are vectors in Π		
	Two correct vectors in the plane	See main scheme	B1
	Eg $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+s\begin{pmatrix}-2\\1\\1\end{pmatrix}+t\begin{pmatrix}1\\-1\\3\end{pmatrix}$		M1
	$x=1-2s+t$ $y=2+s-t$ $z=3+s+3t$	Deduce 3 correct equations	A1
	$4x+7y+z=21$	M1: Eliminate s, t A1: Cao	M1 A1
			(5)

Question	Scheme		Marks
6(b)	AD•AB×AC	Attempt suitable triple product	M1
	$=\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}k-1\\2\\11\end{pmatrix}=4k-4+14+11$		
	$\therefore\frac{1}{6}(4k+21)=6$	M1: Set $\frac{1}{6}$(their triple product) = 6 A1: Correct equation	dM1 A1
	$k=\frac{15}{4}$	Cao (oe)	A1
			(4)
	Alternative		
	Area ABC $=\frac{1}{2}\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert=\frac{1}{2}\sqrt{6}\sqrt{11}$ D to Π is $\frac{4k+28+14-21}{\sqrt{16+49+1}}$	Attempt area *ABC* and distance between *D*and*Π*	M1
	$\frac{1}{6}\sqrt{6}\sqrt{11}\frac{4k+28+14-21}{\sqrt{16+49+1}}=6$	M1: Set $\frac{1}{3}$(their area x their distance) = 6 A1: Correct equation	dM1 A1
	$k=\frac{15}{4}$	Cao (oe)	A1
			(4)
			(9 marks)

Question	Scheme		Marks
7(a)	$x = 3t^4, \quad y = 4t^3$		
	$\frac{\mathrm{d}x}{\mathrm{d}t} = 12t^3, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = 12t^2$	Correct derivatives	B1
	$S = (2\pi)\int y\left(\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2\right)^{\frac{1}{2}} \mathrm{d}t = (2\pi)\int 4t^3\sqrt{(12t^3)^2 + (12t^2)^2}\,\mathrm{d}t$ $\left(= (2\pi)\int 4t^3\left(144t^6 + 144t^4\right)^{\frac{1}{2}} \mathrm{d}t\right)$ M1: Substitutes their derivatives into a correct formula (2π not required)		M1
	$S = (2\pi)\int 4t^3\left(144t^4\right)^{\frac{1}{2}}\left(t^2+1\right)^{\frac{1}{2}} \mathrm{d}t$	Attempt to factor out at least t^4 - numerical factor may be left	M1
	$S = 96\pi\int_0^1 t^5\left(t^2+1\right)^{\frac{1}{2}} \mathrm{d}t$	Correct completion	A1
			(4)
(b)	$u^2 = t^2 + 1 \Rightarrow 2u\frac{\mathrm{d}u}{\mathrm{d}t} = 2t$ or $2u = 2t\frac{\mathrm{d}t}{\mathrm{d}u}$	Correct differentiation	B1
	$t = 0 \Rightarrow u = 1, \; t = 1 \Rightarrow u = \sqrt{2}$	Correct limits **Alternative:** Reverse the substitution later. (Treat as M1 in this case and award later when work seen)	B1
	$S = (96\pi)\int t^5 \times u \times \frac{u}{t}\,\mathrm{d}u$		
	$S = (96\pi)\int \left(u^2-1\right)^2 \times u^2\,\mathrm{d}u$	M1: Complete substitution A1: Correct integral in terms of u. Ignore limits, need not be simplified	M1 A1
	$S = (96\pi)\int \left(u^6 - 2u^4 + u^2\right)\mathrm{d}u = (96\pi)\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]$ M1: Expands and attempts to integrate		dM1
	$S = 96\pi\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]_1^{\sqrt{2}} = 96\pi\left\{\left(\frac{\sqrt{2}^7}{7} - \frac{2\sqrt{2}^5}{5} + \frac{\sqrt{2}^3}{3}\right) - \left(\frac{1}{7} - \frac{2}{5} + \frac{1}{3}\right)\right\}$ M1: Correct use of their changed limits (both to be changed) **Alternative:** If sub reversed, substitute the original limits		ddM1
	$S = \frac{192\pi}{105}\left(11\sqrt{2} - 4\right)$	Cao eg $\frac{64\pi}{35}$	A1
			(7)
			(11 marks)

Question	Scheme		Marks
8(a)	$$I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geq 0$$		
	$\tanh^{2n} x = \tanh^{2(n-1)} x \tanh^2 x$		B1
	$\tanh^{2n} x = \pm \tanh^{2(n-1)} x\left(1-\text{sech}^2 x\right)$		M1
	$I_n = \int_0^{\ln 2} \tanh^{2(n-1)} x \, dx - \int_0^{\ln 2} \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx$		
	$I_n = I_{n-1} - \left[\frac{1}{2n-1}\tanh^{2n-1} x\right]_0^{\ln 2}$	M1: Correctly substitutes for I_{n-1} and obtains $\int \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx = k \tanh^{2n-1} x$ A1: Correct expression	M1 A1
	$= I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$ *	Correct completion with no errors	A1*
			(5)
	Alternative		
	$I_n - I_{n-1} = \int_0^{\ln 2} \left(\tanh^{2n} x - \tanh^{2(n-1)} x\right) dx$		
	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(\tanh^2 x - 1\right) dx$		B1
	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(-\text{sech}^2 x\right) dx$	$= \int_0^{\ln 2} \tanh^{2(n-1)} x\left(\pm\text{sech}^2 x\right) dx$	M1
	$I_n - I_{n-1} = -\left[\frac{1}{2n-1}\tanh^{2n-1} x\right]_0^{\ln 2}$	M1: Obtains $\int \tanh^{2(n-1)} x \, \text{sech}^2 x \, dx = k \tanh^{2n-1} x$ A1: Correct expression	M1 A1
	$= I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$ *	Correct completion with no errors	A1*
			(5)

Question	Scheme		Marks
8(b)	$I_0 = \ln 2$	The integration must be seen.	B1
	$I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	Applies the reduction formula once	M1
	$I_2 = I_0 - \frac{1}{1}\left(\frac{3}{5}\right)^1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	M1: Second application of the reduction formula A1: Correct expression	M1A1
	$I_2 = \ln 2 - \frac{84}{125}$	cao	A1
	Special Case: If I_4 is found award B1 for I_0 or I_1 and M1M0A0A0		
			(5)
	Alternative		
	$I_1 = \int_0^{\ln 2} \tanh^2 x \, dx = \int_0^{\ln 2} \left(1 - \text{sech}^2 x\right) dx$		
	$I_1 = \left[x - \tanh x\right]_0^{\ln 2}$	Correct integration	B1
	$I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	Applies the reduction formula once	M1
	$I_1 = \ln 2 - \tanh(\ln 2) = \ln 2 - \frac{3}{5}$	M1: Uses limits A1: Correct expression	M1A1
	$I_2 = \ln 2 - \frac{3}{5} - \frac{1}{3}\left(\frac{3}{5}\right)^3$		
	$= \ln 2 - \frac{84}{125}$		A1
			(5)
			(10 marks)

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

Sample Assessment Materials for first teaching September 2018
Time: 1 hour 30 minutes

Paper Reference
WME11/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A car is moving along a straight horizontal road with constant acceleration a m s^{-2} ($a > 0$). At time $t = 0$ the car passes the point P moving with speed u m s^{-1}. In the next 4 s, the car travels 76 m and then in the following 6 s it travels a further 219 m.

Find

(i) the value of u,

(ii) the value of a.

(7)

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Question 1 continued

Q1

(Total for Question 1 is 7 marks)

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2. Two particles P and Q are moving in opposite directions along the same horizontal straight line. Particle P has mass m and particle Q has mass km. The particles collide directly. Immediately before the collision, the speed of P is u and the speed of Q is $2u$. As a result of the collision, the direction of motion of each particle is reversed and the speed of each particle is halved.

(a) Find the value of k.

(4)

(b) Find, in terms of m and u only, the magnitude of the impulse exerted on Q by P in the collision.

(2)

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Question 2 continued

Q2

(Total for Question 2 is 6 marks)

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3. A block A of mass 9 kg is released from rest from a point P which is a height h metres above horizontal soft ground. The block falls and strikes another block B of mass 1.5 kg which is on the ground vertically below P. The speed of A immediately before it strikes B is 7 m s^{-1}. The blocks are modelled as particles.

(a) Find the value of h.

(2)

Immediately after the impact the blocks move downwards together with the same speed and both come to rest after sinking a vertical distance of 12 cm into the ground. Assuming that the resistance offered by the ground has constant magnitude R newtons,

(b) find the value of R.

(8)

Question 3 continued

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Question 3 continued

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Question 3 continued

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Q3

(Total for Question 3 is 10 marks)

4.

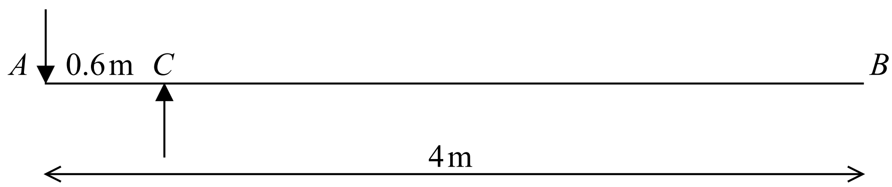

Figure 1

A diving board AB consists of a wooden plank of length 4 m and mass 30 kg. The plank is held at rest in a horizontal position by two supports at the points A and C, where $AC = 0.6$ m, as shown in Figure 1. The force on the plank at A acts vertically downwards and the force on the plank at C acts vertically upwards.

A diver of mass 50 kg is standing on the board at the end B. The diver is modelled as a particle and the plank is modelled as a uniform rod. The plank is in equilibrium.

(a) Find

(i) the magnitude of the force acting on the plank at A,

(ii) the magnitude of the force acting on the plank at C.

(6)

The support at A will break if subjected to a force whose magnitude is greater than 5000 N.

(b) Find, in kg, the greatest integer mass of a diver who can stand on the board at B without breaking the support at A.

(3)

(c) Explain how you have used the fact that the diver is modelled as a particle.

(1)

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Question 4 continued

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Question 4 continued

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Question 4 continued

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5. Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on a particle A.

$\mathbf{F}_1 = (2\mathbf{i} - 3\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N, where p and q are constants.

Given that the resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is parallel to $(\mathbf{i} + 2\mathbf{j})$,

(a) show that $2p - q + 7 = 0$

(5)

Given that $q = 11$ and that the mass of A is 2 kg, and that $\mathbf{F}_1$ and $\mathbf{F}_2$ are the only forces acting on A,

(b) find the magnitude of the acceleration of A.

(5)

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Question 5 continued

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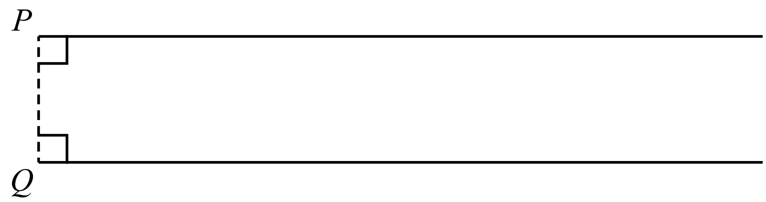

Figure 2

Two cars, A and B, move on parallel straight horizontal tracks. Initially A and B are both at rest with A at the point P and B at the point Q, as shown in Figure 2. At time $t = 0$ seconds, A starts to move with constant acceleration a m s^{-2} for 3.5 s, reaching a speed of 14 m s^{-1}. Car A then moves with constant speed 14 m s^{-1}.

(a) Find the value of a.

(2)

Car B also starts to move at time $t = 0$ seconds, in the same direction as car A. Car B moves with a constant acceleration of 3 m s^{-2}. At time $t = T$ seconds, B overtakes A. At this instant A is moving with constant speed.

(b) On a diagram, sketch, on the same axes, a speed-time graph for the motion of A for the interval $0 \leqslant t \leqslant T$ and a speed-time graph for the motion of B for the interval $0 \leqslant t \leqslant T$.

(3)

(c) Find the value of T.

(8)

(d) Find the distance of car B from the point Q when B overtakes A.

(1)

(e) On a new diagram, sketch, on the same axes, an acceleration-time graph for the motion of A for the interval $0 \leqslant t \leqslant T$ and an acceleration-time graph for the motion of B for the interval $0 \leqslant t \leqslant T$.

(3)

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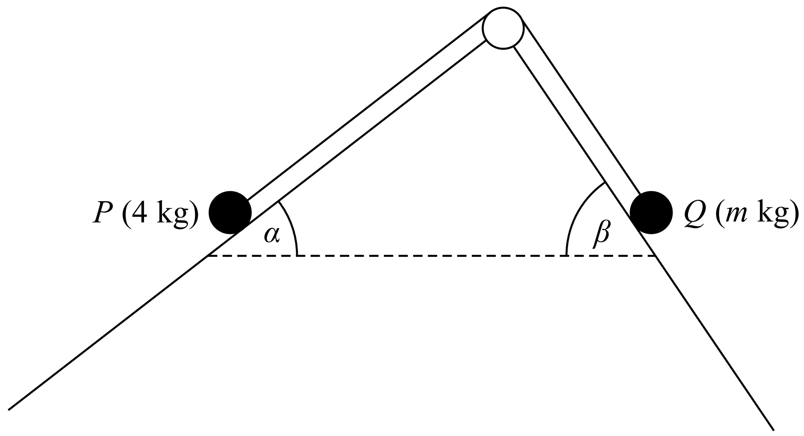

Figure 3

A particle P of mass 4 kg is attached to one end of a light inextensible string. A particle Q of mass m kg is attached to the other end of the string. The string passes over a small smooth pulley which is fixed at a point on the intersection of two fixed inclined planes. The string lies in a vertical plane that contains a line of greatest slope of each of the two inclined planes. The first plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$ and the second plane is inclined to the horizontal at an angle β, where $\tan\beta = \frac{4}{3}$. Particle P is on the first plane and particle Q is on the second plane with the string taut, as shown in Figure 3.

The first plane is rough and the coefficient of friction between P and the plane is $\frac{1}{4}$. The second plane is smooth. The system is in limiting equilibrium.

Given that P is on the point of slipping down the first plane,

(a) find the value of m,

(10)

(b) find the magnitude of the force exerted on the pulley by the string,

(4)

(c) find the direction of the force exerted on the pulley by the string.

(1)

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Question 7 continued

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Question 7 continued

Q7

(Total for Question 7 is 15 marks)

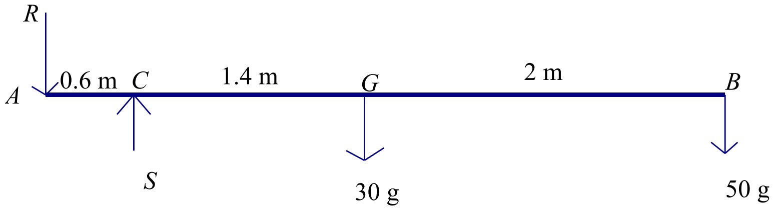
TOTAL FOR PAPER IS 75 MARKS

Mechanics M1 Mark scheme

Question	Scheme		Marks
1	$76 = 4u + \frac{1}{2}a.4^2$ or $76 = \frac{1}{2}\left(u + \overline{u + 4a}\right) \times 4$	Use of $s = ut + \frac{1}{2}at^2$ for $t = 4, s = 76$ and $u \neq 0$ (use of $u = 0$ is M0)	M1
	$(38 = 2u + 4a)$	Correctly substituted equation	A1
	$295 = 10u + \frac{1}{2}a.10^2$ or $295 = \frac{1}{2}\left(u + \overline{u + 10a}\right) \times 10$ or $295 = (u + 10a) \times 10 - \frac{1}{2}a \times 100$	Use of $s = ut + \frac{1}{2}at^2$ for $t = 10, s = 295$ or $s = u't + \frac{1}{2}at^2$ for $t = 6, s = 219, u' \neq u$	M1
	$(59 = 2u + 10a)$ or $219 = (19 + 2a) \times 6 + \frac{1}{2}a \times 6^2$ or $219 = (38 - u) \times 6 + \frac{1}{2}a \times 6^2$ or $219 = (u + 4a) \times 6 + \frac{1}{2}a \times 6^2$ or $219 = \frac{1}{2}\left(\overline{u + 4a} + \overline{u + 10}\right) \times 6$ or $219 = (u + 10a) \times 6 - \frac{1}{2}a \times 36$	Correctly substituted equation	A1
	Solve simultaneous for u or for a. This marks is not available if they have assumed a value for u or a in the preceding work - it is dependent on the first 2 M marks.		DM1
	$u = 12$		A1
	$a = 3.5$		A1
			(7)
	Alternative		
	$t = 2,\ v_2 = \frac{76}{4} = 19$ $t = 7,\ v_7 = \frac{219}{6} = 36.5$	Find the speed at $t = 2, t = 7$ Both values correct Averages with no links to times is M0	M1 A1
	$36.5 = 19 + 5a \Rightarrow a = 3.5$	Use of $v = u + 5a$ with their u,v Correct a	M1 A1
	$19 = u + 2a$	Complete method for finding u Correct equation in u	DM1 A1
	$u = 19 - 7 = 12$		A1
			(7)
			(7 marks)

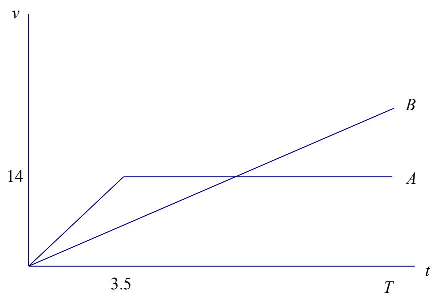
Question	Scheme		Marks
2(a)	$mu - 2kmu = -\frac{1}{2}mu + kmu$ or $m\left(\frac{1}{2}u + u\right) = -km\left(-u - 2u\right)$	Use of CLM or Equal and opposite impulses Need all 4 terms dimensionally correct. Masses and speeds must be paired correctly Condone sign errors Condone factor of g throughout.	M1
	Unsimplified equation with at most one error		A1
	Correct unsimplified equation		A1
	$k = \frac{1}{2}$	From correct working only	A1
			(4)
(b)	For $P: I = \pm m(\frac{1}{2}u \pm -u)$ For $Q: I = \pm km(u \pm -2u)$	Impulse on P or impulse on Q. Mass must be used with the correct speeds e.g. $km \times \frac{1}{2}u$ is M0 If working on Q, allow equation using their k. Terms must be dimensionally correct. Use of g is M0	M1
	$\frac{3mu}{2}$	Only From correct working only	A1
			(2)
			(6 marks)

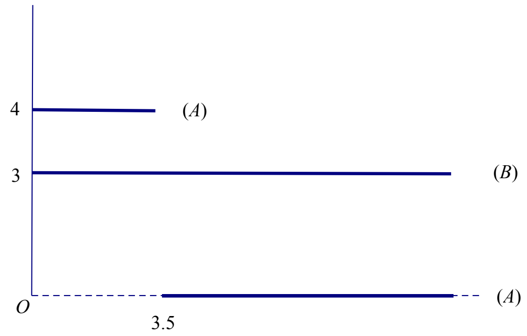
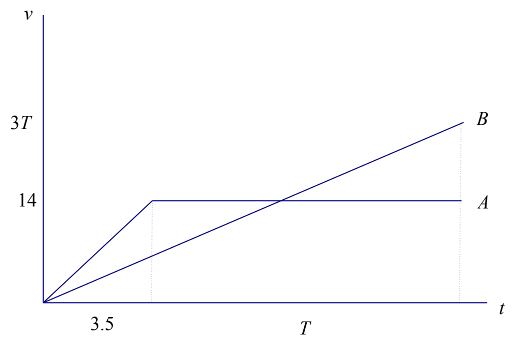
Question	Scheme		Marks
3(a)	$7^2 = 2 \text{ x } 9.8h$	Use of $v^2 = u^2 + 2as$ with $u = 0, v = 7$ or alternative complete method to find h	M1
	$h = 2.5$	Condone $h = -2.5$ in the working but the final answer must be positive.	A1
			(2)
(b)	$9 \text{ x } 7 = 10.5\,u$	Use CLM to find the speed of the blocks after the impact. Condone additional factor of g throughout.	M1
	$u = 6$		A1
	$0^2 = 6^2 - 2a \text{ x } 0.12$	Use of $v^2 = u^2 + 2as$ with $u = 6, v = 0$ Allow for their u and $v = 0$ Allow for $u = 7, v = 0$ Accept alternative *suvat* method to form an equation in a. Condone use of 12 for 0.12	M1
		Correctly substituted equation in a with $u = 6, s = 0.12$ (implied by $a = 150$)	A1
	$(\downarrow)$ $10.5g - R = 10.5 \text{ x } (\text{-}a)$	Use of $F = ma$ with their $a \neq \pm$g . Must have all 3 terms and 10.5 Condone sign error(s)	M1
	$(\downarrow)$ $10.5g - R = 10.5 \text{ x } (\text{-}150)$	Unsimplified equation with a substituted and at most one error (their a with the wrong sign is 1 error)	A1
		Correct unsimplified equation with a substituted	A1
	$R = 1680$ or 1700		A1
			(8)
	Alternative for the last 6 marks:		
	$\frac{1}{2} \times 10.5 \times 6^2 + 10.5 \times 9.8 \times 0.12 = R \times 0.12$	Energy equation (needs all three terms)	M2
		-1 each error A1A1A0 for 1 error, A1A0A0 for 2 errors	A3
	$R = 1680$ or 1700		A1

Question	Scheme		Marks
4(a)			
	$M(A)$ (30g x 2) + (50g x 4) = 0.6 S	Moments equation. Requires all terms and dimensionally correct. Condone sign errors. Allow M1 if g missing	M1
	$M(C)$ $(0.6\times R)=(1.4\times 30\text{g})+(3.4\times 50\text{g})$ $M(G)$ $(2\times R)=(1.4\times S)+(2\times 50\text{g})$ $M(B)$ $(4\times R)+(2\times 30g)=(3.4\times S)$	Correct unsimplified equation	A1
	$(\uparrow)$ $R+30g+50g=S$ $(R+784=S)$	Resolve vertically. Requires all 4 terms. Condone sign errors	M1
	Correct equation (with R or their R) NB: The second M1A1 can also be earned for a second moments equation		A1
	$R=3460$ or 3500 or $\frac{1060g}{3}$ (N) Not 353.3g	One force correct	A1
	$S=4250$ or 4200 or $\frac{1300g}{3}$ (N) Not 433.3g	Both forces correct If both forces are given as decimal multiples of g mark this as an accuracy penalty A0A1	A1
			(6)
(b)	$M(C)$ (30g x 1.4) + (Mg x 3.4) = 0.6 x 5000	Use $R=5000$ and complete method to form an equation in M or weight. Needs all terms present and dimensionally correct. Condone sign errors. Accept inequality. Use of R and S from (a) is M0	M1
		Correct equation in M (not weight) (implied by $M=77.68$)	A1
	$M=77$ kg	77.7 is A0 even is the penalty for over-specified answers has already been applied	A1
			(3)

Question	Scheme		Marks
4(c)	The weight of the diver acts at a point.	Accept “the mass of the diver is at a point”.	B1
			(1)
			(10 marks)

Question	Scheme		Marks
5(a)	$(2\mathbf{i}-3\mathbf{j})+(p\mathbf{i}+q\mathbf{j})=(p+2)\mathbf{i}+(q-3)\mathbf{j}$	Resultant force = $\mathbf{F}_1+\mathbf{F}_2$ in the form $a\mathbf{i}+b\mathbf{j}$	M1
	$\frac{p+2}{q-3}=\frac{1}{2}$ or $\left.\begin{array}{l}p+2=n\\ q-3=2n\end{array}\right\}$ for $n\neq 1$	Use parallel vector to form a scalar equation in p and q.	M1
		Correct equation (accept any equivalent form)	A1
	$4+2p=-3+q$	Dependent on no errors seen in comparing the vectors. Rearrange to obtain given answer. At least one stage of working between the fraction and the given answer	DM1
	$2p-q+7=0$	**Given Answer**	A1
			(5)
5(b)	$q=11\Rightarrow p=2$		B1
	$\mathbf{R}=4\mathbf{i}+8\mathbf{j}$	$(2+p)\mathbf{i}+8\mathbf{j}$ for their p	M1
	$4\mathbf{i}+8\mathbf{j}=2\mathbf{a}$ $(\mathbf{a}=2\mathbf{i}+4\mathbf{j})$	Use of $\mathbf{F}=m\mathbf{a}$	M1
	$\lvert\mathbf{a}\rvert=\sqrt{2^2+4^2}$	Correct method for $\lvert\mathbf{a}\rvert$ Dependent on the preceding M1	DM1
	$=\sqrt{20}=4.5$ or 4.47 or better $(\mathrm{m\ s^{-2}})$	$2\sqrt{5}$	A1
			(5)
	Alternative for the last two M marks:		
	$\lvert\mathbf{F}\rvert=\sqrt{16+64}\left(=\sqrt{80}\right)$	Correct method for $\lvert\mathbf{F}\rvert$	M1
	$\sqrt{80}=2\times\lvert\mathbf{a}\rvert$	Use of $\lvert\mathbf{F}\rvert=m\lvert\mathbf{a}\rvert$ Dependent on the preceding M1	DM1
			(5)
			(10 marks)

Question	Scheme		Marks
6(a)	$v = u + at \Rightarrow 14 = 3.5a$	Use of *suvat* to form an equation in a	M1
	$a = 4$		A1
			(2)
(b)		Graph for A or B	B1
		Second graph correct and both graphs extending beyond the point of intersection	B1
		Values 3.5, 14, T shown on axes, with T not at the point of intersection. Accept labels with delineators.	B1
	NB: 2 separate diagrams scores max B1B0B1		**(3)**
(c)	$\frac{1}{2}T.3T$, $\frac{(T + T - 3.5)}{2}.14$	Find distance for A or B in terms of T only. Correct area formulae: must see $\frac{1}{2}$ in area formula and be adding in trapezium	M1
	One distance correct		A1
	Both distances correct		A1
	$\frac{1}{2}T.3T = \frac{(T + T - 3.5)}{2}.14$ $\frac{1}{2}T.3T = \frac{1}{2} \times 4 \times 3.5^2 + 14(T - 3.5)$	Equate distances and simplify to a 3 term quadratic in T in the form $aT^2 + bT + c = 0$	M1
	$3T^2 - 28T + 49 = 0$	Correct quadratic	A1
	$(3T - 7)(T - 7) = 0$	Solve 3 term quadratic for T	M1
	$T = \frac{7}{3}$ or 7	Correct solution(s) - can be implied if only ever see $T = 7$ from correct work.	A1
	but $T > 3.5$, $T = 7$		A1
			(8)
(d)	73.5 m	From correct work only. B0 if extra answers.	B1
			(1)

Question	Scheme		Marks
6(e)		(A) Condone missing 4	B1
		(B) Condone graph going beyond $T=7$ Must go beyond 3.5. Condone no 3.	B1
		(A) Condone graph going beyond $T=7$ Must go beyond 3.5. B0 if see a <u>solid</u> vertical line. Sometimes very difficult to see. If you think it is there, give the mark.	B1
			(3)
	Condone separate diagrams.		
	Alternative for (c) for candidates with a sketch like this: 	Treat as a special case. B1B1B0 on the graph and then max 5/8 for (c) if they do not solve for the T in the question.	B1 B1 B0
	$\frac{1}{2}\times 3\times(T+3.5)^2=\frac{1}{2}\times 4\times 3.5^2+14T$	Use diagram to find area	M1
		One distance correct	A1
		Both distances correct	A1
	$12T^2-28T-49=0$	Simplify to a 3 term quadratic in T	M1
		Correct quadratic	A1
	$(2T-7)(6T+7)=0$	Complete method to solve for the T in the question	M1
	$T=\frac{7}{2}$ or $\frac{-7}{6}$	Correct solution(s) - can be implied if only ever see Total $=7$	A1
	Total time $=7$		A1
			(8)
			(17 marks)

Question	Scheme		Marks
7(a)	$F = 0.25R$		B1
	$\sin\alpha = \frac{3}{5}$ or $\cos\alpha = \frac{4}{5}$ $\sin\beta = \frac{4}{5}$ or $\cos\beta = \frac{3}{5}$	Use of correct trig ratios for α or β	B1
	$R = 4g\cos\alpha$ (31.36)	Normal reaction on P Condone trig confusion (using α)	M1
		Correct equation	A1
	$T + F = 4g\sin\alpha$	Equation of motion for P. Requires all 3 terms. Condone consistent trig confusion Condone an acceleration not equated to 0 : $T + F - 4g\sin\alpha = 4a$	M1
	$(T + 7.84 = 23.52)$ $(T = 15.68)$	Correct equation	A1
	$T = mg\sin\beta$	Equation of motion for Q Condone trig confusion Condone an acceleration not equated to 0: $T - mg\sin\beta = -ma$	M1
	$(T = 7.84m)$	Correct equation	A1
	Solve for m	Dependent on the 3 preceding M marks Not available if their equations used $a \neq 0$	DM1
	$m = 2$		A1
	NB Condone a whole system equation $4g\sin\alpha - F = mg\sin\beta$ followed by $m = 2$ for 6/6 M2 for an equation with all 3 terms. Condon trig confusion. Condone an acceleration $\neq 0$ A2 (-1 each error) for a correct equation:		
			(10)
7(b)	$F = \sqrt{T^2 + T^2}$ or $2T\cos 45°$ or $\dfrac{T}{\cos 45}$	Complete method for finding F in terms of T Accept $\sqrt{(R_h)^2 + (R_v)^2}$	M1
	Correct expression in T		A1
	Substitute their T into a correct expression. Dependent on the previous M mark		DM1
	$F = \sqrt{2}\dfrac{8g}{5} = 22$ or 22.2 (N)	Watch out - resolving vertically is not a correct method and gives 21.9 N.	A1
			(4)

Question	Scheme		Marks
7(c)	Along the angle bisector at the pulley	Or equivalent - accept angle + arrow shown on diagram. ($8.1°$ to downward vertical) Do not accept a bearing	
			(1)
			(15 marks)

Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Mathematics
International Advanced Subsidiary/Advanced Level
Mechanics M2

Sample Assessment Materials for first teaching September 2018
Time: 1 hour 30 minutes

Paper Reference
WME12/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1. A car of mass 900 kg is travelling up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{25}$. The car is travelling at a constant speed of $14\,\text{m s}^{-1}$ and the resistance to motion from non-gravitational forces has a constant magnitude of 800 N. The car takes 10 seconds to travel from A to B, where A and B are two points on the road.

(a) Find the work done by the engine of the car as the car travels from A to B.

(4)

When the car is at B and travelling at a speed of $14\,\text{m s}^{-1}$ the rate of working of the engine of the car is suddenly increased to P kW, resulting in an initial acceleration of the car of $0.7\,\text{m s}^{-2}$. The resistance to motion from non-gravitational forces still has a constant magnitude of 800 N.

(b) Find the value of P.

(4)

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Question 1 continued

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Question 1 continued

Q1

(Total for Question 1 is 8 marks)

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2. A particle P of mass 0.7 kg is moving in a straight line on a smooth horizontal surface. The particle P collides with a particle Q of mass 1.2 kg which is at rest on the surface. Immediately before the collision the speed of P is $6\,\mathrm{m\,s^{-1}}$. Immediately after the collision both particles are moving in the same direction. The coefficient of restitution between the particles is e.

(a) Show that $e < \dfrac{7}{12}$

(7)

Given that $e = \dfrac{1}{4}$

(b) find the magnitude of the impulse exerted on Q in the collision.

(3)

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Question 2 continued

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3. At time t seconds ($t \geqslant 0$) a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (6t^2 + 6t)\mathbf{i} + (3t^2 + 24)\mathbf{j}$$

When $t = 0$ the particle P is at the origin O. At time T seconds, P is at the point A and $\mathbf{v} = \lambda(\mathbf{i} + \mathbf{j})$, where λ is a constant.

Find

(a) the value of T,

(3)

(b) the acceleration of P as it passes through the point A,

(3)

(c) the distance OA.

(5)

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Question 3 continued

Q3

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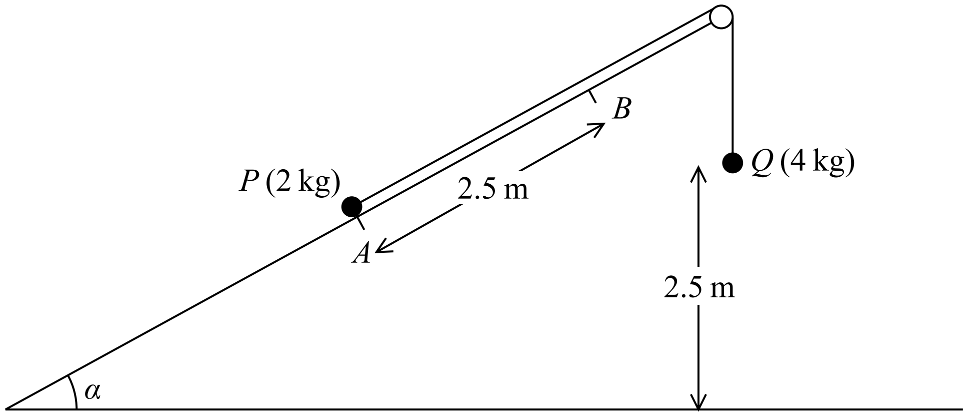

Figure 1

Two particles P and Q, of mass 2 kg and 4 kg respectively, are connected by a light inextensible string. Initially P is held at rest at the point A on a rough fixed plane inclined at α to the horizontal ground, where $\sin\alpha = \frac{3}{5}$. The string passes over a small smooth pulley fixed at the top of the plane. The particle Q hangs freely below the pulley and 2.5 m above the ground, as shown in Figure 1. The part of the string from P to the pulley lies along a line of greatest slope of the plane. The system is released from rest with the string taut. At the instant when Q hits the ground, P is at the point B on the plane. The coefficient of friction between P and the plane is $\frac{1}{4}$.

(a) Find the work done against friction as P moves from A to B.

(4)

(b) Find the total potential energy lost by the system as P moves from A to B.

(3)

(c) Find, using the work-energy principle, the speed of P as it passes through B.

(4)

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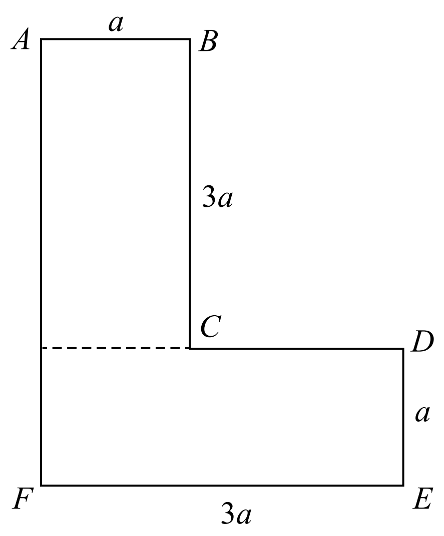

Figure 2

The uniform lamina *ABCDEF*, shown in Figure 2, consists of two identical rectangles with sides of length a and $3a$. The mass of the lamina is M. A particle of mass kM is attached to the lamina at E. The lamina, with the attached particle, is freely suspended from A and hangs in equilibrium with AF at an angle θ to the downward vertical.

Given that $\tan\theta = \frac{4}{7}$, find the value of k.

(10)

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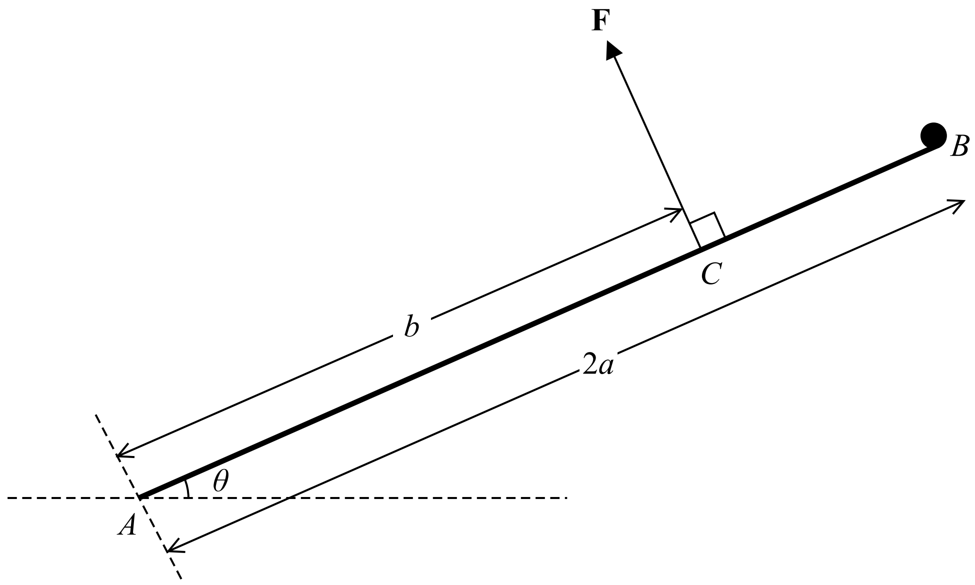

Figure 3

A uniform rod AB, of mass $3m$ and length $2a$, is freely hinged at A to a fixed point on horizontal ground. A particle of mass m is attached to the rod at the end B. The system is held in equilibrium by a force **F** acting at the point C, where $AC = b$. The rod makes an acute angle θ with the ground, as shown in Figure 3. The line of action of **F** is perpendicular to the rod and in the same vertical plane as the rod.

(a) Show that the magnitude of **F** is $\dfrac{5mga}{b}\cos\theta$

(4)

The force exerted on the rod by the hinge at A is **R**, which acts upwards at an angle ϕ above the horizontal, where $\phi > \theta$.

(b) Find

(i) the component of **R** parallel to the rod, in terms of m, g and θ,

(ii) the component of **R** perpendicular to the rod, in terms of a, b, m, g and θ.

(5)

(c) Hence, or otherwise, find the range of possible values of b, giving your answer in terms of a.

(2)

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Question 6 continued

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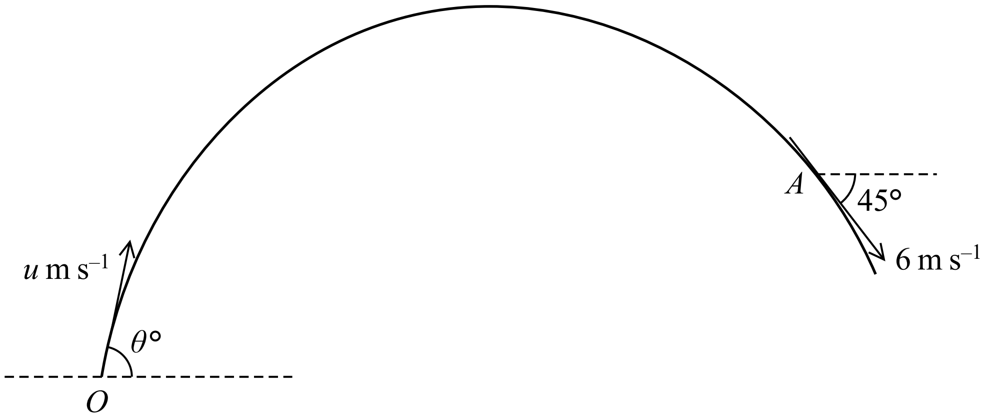

Figure 4

At time $t = 0$, a particle P of mass 0.7 kg is projected with speed u m s^{-1} from a fixed point O at an angle $\theta°$ to the horizontal. The particle moves freely under gravity. At time $t = 2$ seconds, P passes through the point A with speed 6 m s^{-1} and is moving downwards at 45° to the horizontal, as shown in Figure 4.

Find

(a) the value of θ,

(6)

(b) the kinetic energy of P as it reaches the highest point of its path.

(3)

For an interval of T seconds, the speed, v m s^{-1}, of P is such that $v \leqslant 6$

(c) Find the value of T.

(5)

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Question 7 continued

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(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS

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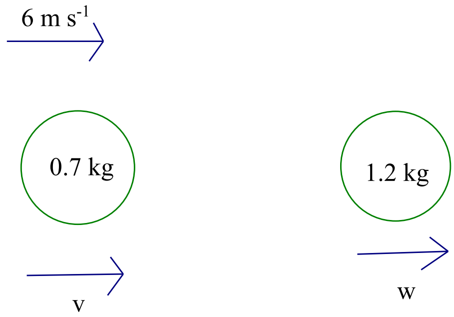
Mechanics M2 Mark scheme

Question	Scheme		Marks
1(a)	Resolving parallel to the plane	Condone trig confusion	M1
	$D = 900g\sin\theta + 800$		A1
	$\frac{900}{25}g + 800(= 1152.8)$ (N)		
	Work done : Their $D \times$ distance $= 1152.8 \times 14 \times 10$	Independent. For use of 14 x 10 x their D	M1
	$= 161392 = 161$ kJ (160)	Accept 161000 (J), 160000 (J). Ignore incorrect units.	A1
			(4)
	Alternative using energy		
	Work done $= 900gd\sin\theta + 800d$	Allow with incorrect d	M1A1
	Use of $d = 14 \times 10$	Independent – allow in an incorrect expression	M1
	$= 161392 = 161$ kJ (160)		A1
			(4)
1(b)	Equation of motion	All terms required. Condone trig confusion and sign errors. Allow with $900a$	M1
	$D - 900g\sin\theta - 800 = 900 \times 0.7$	Correct unsimplified with $a = 0.7$ used. Accept with their 1152.8 arising from a 2 term expression in (a)	A1
	($D - 1152.8 = 900 \times 0.7$)		
	$D = 1782.8$ (N)		
	Use of $P = Fv$ $\quad P = 14 \times \frac{\text{their } D}{1000}$	Independent. Treat missing 1000 as misread, so allow for $14 \times \text{their} D$. Allow for $\frac{1000P}{14}$ (or $\frac{P}{14}$) in their equation of motion	M1
	$P = 25.0$ (25)	cao	A1
			(4)
			(8 marks)

Question	Scheme		Marks
2(a)			
	CLM: $0.7\times 6=0.7\times v+1.2w$	Requires all terms & dimensionally correct	M1
	$(42=7v+12w)$	Correct unsimplified	A1
	Impact:	Used the right way round Condone sign errors	M1
	$w-v=6e$		A1
	Equation in e and v only: $42-72e=19v$	Dependent on the two previous M marks	DM1
	Use direction to form an inequality:	Independent. Applied correctly for their v	M1
	$42-72e>0 \quad \Rightarrow \quad e<\frac{7}{12}$	***Given answer***	A1
			(7)
2(b)	Impulse on Q: $I=w\times 1.2$		M1
	Solve for w : $w=v+6e=\dfrac{42-72\times\frac{1}{4}}{19}+6\times\frac{1}{4}$	Accept unsimplified with e substituted. Have to be using w in part (b) $w=\frac{105}{38}=2.763.....$ seen or implied	B1
	$I=1.2\times\frac{42}{19}\times\frac{5}{4}=\frac{63}{19}(=3.32)$ (N s)	3.3 or better	A1
			(3)
	Alternative		
	Impulse on Q = – impulse on P		
	$=-0.7(v-6)$	Accept negative here	M1
	$=-0.7\left(\dfrac{42-\frac{1}{4}\times 72}{19}-6\right)$	Substitute for e in their v $v=\frac{24}{19}=1.263....$ seen or implied Accept negative here.	B1

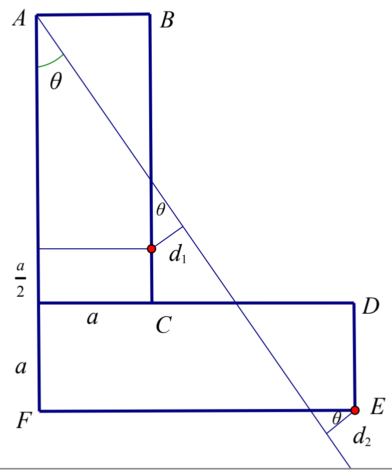
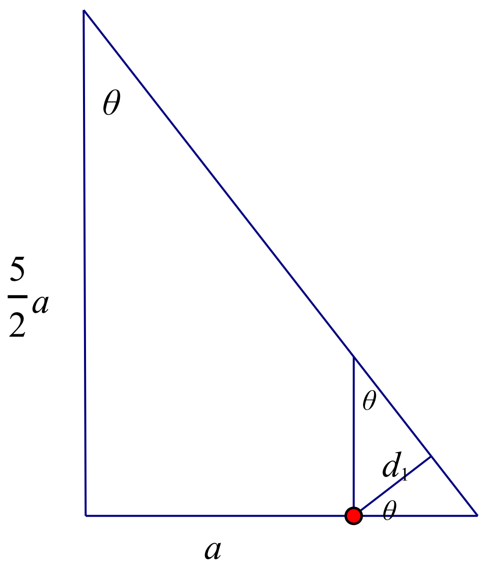
Question	Scheme		Marks
2(b) ***continued***	$=\dfrac{63}{19}$	Final answer must be positive. 3.3 or better	A1
			(3)
			(10 marks)

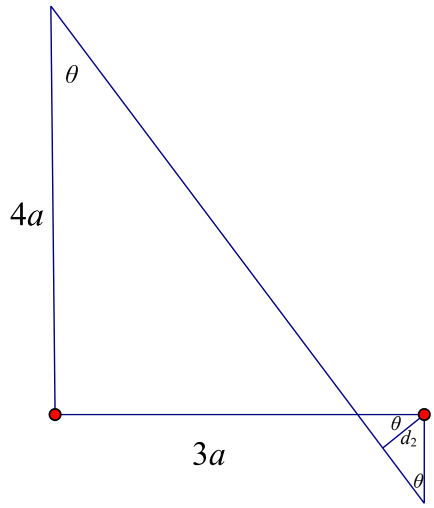
Question	Scheme		Marks
3(a)	Use $\mathbf{v} = \lambda(\mathbf{i}+\mathbf{j})$: $6T^2 + 6T = 3T^2 + 24$	Form an equation in t, T or λ $\lambda^2 - 108\lambda + 2592 = 0$	M1
	Solve for T $\quad 3T^2 + 6T - 24 = 0$,	Simplify to quadratic in t, T or λ and solve.	M1
	$(T+4)(T-2) = 0$, $T = 2$	$T = 2$ only	A1
	If they score M1 and then state $T = 2$ allow 3/3		
	If they guess $T = 2$ and show that it works then allow 3/3.		
	If all we see is $T = 2$ with no equation then 0/3 for (a) but full marks are available for (b) and (c).		
			(3)
3(b)	Differentiate: $\mathbf{a} = (12t+6)\mathbf{i} + 6t\mathbf{j}$	Majority of powers going down Need to be considering both components	M1
		Correct in t or T	A1
	$= 30\mathbf{i} + 12\mathbf{j}$ (m s^{-2})	Cao	A1
			(3)
3(c)	Integrate : $\mathbf{r} = \left(2t^3 + 3t^2(+A)\right)\mathbf{i} + \left(t^3 + 24t(+B)\right)\mathbf{j}$	Clear evidence of integration. Need to be considering both components. Do not need to see the constant(s).	M1
	-1 each error		A2
	If the integration is seen in part (a) it scores no marks at that stage, but if the result is used in part (c) then the M1A2 is available in part (c)		
	$\boldsymbol{OA} = 28\mathbf{i} + 56\mathbf{j}$ $\quad$ Use their T		
	Distance $= 28\sqrt{5} = 62.6$ (m)	Dependent on previous M1 Use of Pythagoras on their $\boldsymbol{OA}$	DM1
	63 or better , $\sqrt{3920}$		A1
	NB: Incorrect T can score 2/3 in (b) and 4/5 in (c)		
			(5)
			(11 marks)

Question	Scheme		Marks
4(a)	Resolve perpendicular to the plane: $R = 2g\cos\alpha$		B1
	Use $F = \mu R$: $F = \frac{1}{4}\times 2g\times\frac{4}{5}\left(=\frac{2g}{5}\right)$	with $\frac{1}{4}$ and their R (3.92)	M1
	Work done: WD $= 2.5\times F$	For their F	dM1
	$= 2.5\times\frac{2g}{5} = 9.8$ (J)	Accept g	A1
	If a candidate has found the total work done but you can see the correct terms/processes for finding the work done against friction, give B1M1DM1A0 (3/4)		
			(4)
4(b)	Change in PE : $\pm\left(4g\times 2.5 - 2g\times 2.5\sin\alpha\right)$	Requires one gaining and one losing Condone trig confusion	M1
	$=\pm\left(4g\times 2.5 - 2g\times 1.5\right)$	$\pm$ (correct unsimplified)	A1
	PE lost $= 7g = 68.6$ (J)	or 69 (J) Accept $7g$	A1
			(3)
4(c)	KE gained + WD = loss in GPE	The question requires the use of work-energy. Alternative methods score 0/4. Requires all terms but condone sign errors (must be considering both particles)	M1
	$\frac{1}{2}\times 4v^2 + \frac{1}{2}\times 2v^2 + \left(\text{their (a)}\right) = \left(\text{their (b)}\right)$	Correct unsimplified. -1 each error	A2
	$3v^2 = 6g$		
	$v = \sqrt{2g} = 4.43$ (m s^{-1})	or 4.4. Accept $\sqrt{2g}$	A1
			(4)
	Alternative		
	Equations of motion for each particle leading to $T = \frac{12g}{5} = 23.52$ followed by a W-E equation for P: $2.5T = \frac{1}{2}\times 2v^2 + 2g\times 2.5\sin\alpha + (a)$ M1A2	Equations of motion for each particle leading to $T = \frac{12g}{5} = 23.52$ followed by a W-E equation for Q: $\frac{1}{2}\times 4v^2 + 2.5T = 4g\times 2.5$	
	$v = \sqrt{2g} = 4.43$ (m s^{-1})		A1

Question	Scheme	Marks
4(c) ***continued***	Use of $\alpha = 36.9$ gives correct answers to 3 sf	
	Use of $\alpha = 37$ gives correct answers to 2 sf and more than this is not justified, so A0 if they give 3 sf in this case.	
		(11 marks)

Question	Scheme		Marks
5	Moments about **vertical** axis (AF):	Requires all terms and dimensionally correct but condone g missing	M1
	$\frac{Mg}{2}\times\frac{1}{2}a+\frac{Mg}{2}\times1.5a+3akMg=Mg(1+k)\bar{x}$	-1 each error Accept with M and/or g not seen.	A2
	$\left(\bar{x}=\frac{1+3k}{1+k}a\right)$		
	Moments about **horizontal** axis (AB or FE):	Requires all terms and dimensionally correct but condone g missing	M1
	$\frac{Mg}{2}\times1.5a+\frac{Mg}{2}\times3.5a+4akMg=Mg(1+k)\bar{y}$	-1 each error. Accept with M and/or g not seen. Do not penalise repeated errors.	A2
	$\left(\bar{y}=\frac{2.5+4k}{1+k}a\right)$		
		Working with axes through F gives $\bar{x}=\frac{1+3k}{1+k}a$ and $\bar{y}=\frac{1.5}{1+k}a$	
	SR: A candidate working with a mixture of mass and mass ratio can score 4/6 M1A0A0M1A2		
	Use of $\tan\theta$ with their distances from AF & AB	Must be considering the whole system. Allow for inverted ratio.	M1
	$\tan\theta=\frac{M+3kM}{2.5M+4kM}\left(=\frac{4}{7}\right)$	or exact equivalent	A1
	Equate their $\tan\theta$ to $\frac{4}{7}$ and solve for k : $7M+21kM=10M+16kM$		M1
	$k=\frac{3}{5}$	cso	A1
			(10)
	Alternative for the people who start by considering only the L shape.		

Question	Scheme		Marks
5 ***continued***	$\bar{x} = a$ and $\bar{y} = \frac{5}{2}a$ or $\frac{3}{2}a$	M1 (for either) requires all terms and dimensionally correct but condone $g\,/M$ missing. A1 for each correct.	M1A2
	Combine with the particle	M1 (for both) requires all terms and dimensionally correct but condone g missing. A1 for each correct.	M1A2
	See over for a more geometrical approach		
		Candidate starts by finding centre of mass at $\left(a, \frac{3}{2}a\right)$ relative to F (or equivalent), M1A2 scored	
		Use of $\tan\theta$ with their distances for finding d_1 or d_2.	M1
		Obtain length of a side in a triangle containing d_1 $\left(\frac{5}{2}a\right)\tan\theta - a\left(= \frac{3}{7}a\right)$ Correct for their centre of mass	A1

Question	Scheme		Marks
5 ***continued***		$d_1 = \left(\frac{3}{7}a\right)\cos\theta$ Correct for their centre of mass	A1
		Use of $\tan\theta$ to find second distance $3a - 4a\tan\theta = \frac{5}{7}a$	M1
		$d_2 = \frac{5}{7}a\cos\theta$	A1
	Moments about A: $Md_1 = kMd_2$		M1
	$\frac{3}{7}a\cos\theta = k \times \frac{5}{7}a\cos\theta \;\Rightarrow\; k = \frac{3}{5}$		A1
			(10)
			(10 marks)

Question	Scheme		Marks
6(a)	Taking moments about A:	Requires all terms - condone trig confusion and sign errors	M1
	$bF = 3mga\cos\theta + mg \times 2a\cos\theta$	-1 each error	A2
	$bF = 5mga\cos\theta$ $F = \frac{5mga}{b}\cos\theta$	***Given answer***	A1
			(4)
6(b)	Component of **R** parallel to AB: $\left(R\cos(\phi-\theta)\right)$	Requires all terms - condone trig confusion	M1
	$= 3mg\sin\theta + mg\sin\theta = 4mg\sin\theta$	Correct unsimplified	A1
	Component of **R** perpendicular to AB:	Requires all terms - condone consistent trig confusion and sign errors	M1
	$\left(R\sin(\phi-\theta)\right) + F = 4mg\cos\theta$	Correct unsimplified	A1
	Alternatives for: $M(B)$	$2aR\sin(\phi-\theta) + 3mga\cos\theta = F(2a-b)$	M1A1
	$M(C)$	$bR\sin(\phi-\theta) + (2a-b)mg\cos\theta$ $= 3mg(b-a)\cos\theta$	
	$\left(R\sin(\phi-\theta)\right) = 4mg\cos\theta - \frac{5mga}{b}\cos\theta$	Correct with F substituted.	A1
	ISW for incorrect work after correct components seen		**(5)**
	Alternative		
	$X = F\sin\theta = \frac{5mga}{b}\cos\theta\sin\theta$	Allow with F. Requires all terms - condone trig confusion	M1
	F substituted		A1
	$Y = 4mg - F\cos\theta = 4mg - \frac{5mga}{b}\cos^2\theta$	Allow with F. Requires all terms - condone trig confusion and sign errors.	M1
	Correct unsimplified		A1
	Correct substituted		A1
			(5)
6(c)	Use of $R\sin(\phi-\theta) > 0$		M1
	Solve for b in terms of a: $4 > \frac{5a}{b}$, $(2a \geq)b > \frac{5}{4}a$	$2a$ not required CSO	A1
			(2)
	Special case:		
	Misread of directions in (b)	**NB:** This MR can score full marks	**(2)**

Question	Scheme		Marks
6(c) *continued*	**Alternative**		
	For $\varphi > \theta, \tan\phi > \tan\theta$		
	$\tan\varphi = \frac{Y}{X} = \frac{4 - \frac{5a}{b}\cos^2\theta}{\frac{5a}{b}\cos\theta\sin\theta} > \tan\theta$		M1
	$4 - \frac{5a}{b}\cos^2\theta > \frac{5a}{b}\sin^2\theta$		
	$4 > \frac{5a}{b}\left(\cos^2\theta + \sin^2\theta\right) \quad \Rightarrow b > \frac{5}{4}a$	cso	A1
			(2)
			(11 marks)

Question	Scheme		Marks
7(a)	Equate horizontal components of speeds:		M1
	$u\cos\theta° = 6\cos 45°\left(=3\sqrt{2}\right)\ (4.24....)$	Correct unsimplified	A1
	Use suvat for vertical speeds: $u\sin\theta° - 2g = -6\sin 45°$	Condone sign errors	M1
	$\left(u\sin\theta = 2g - 3\sqrt{2}\right)$	Correct unsimplified	A1
	Divide to find $\tan\theta$: $\tan\theta = \dfrac{2g - 6\sin 45}{6\cos 45}$	Dependent on previous 2 Ms. Follow their components.	DM1
	$\left(= \dfrac{2g - 3\sqrt{2}}{3\sqrt{2}} = 3.61..\right) \Rightarrow$ $\theta = 74.6\ (75)$	$(u = 15.93....)$	A1
			(6)
7(b)	At max height, speed $= u\cos\theta (= 3\sqrt{2}\ (\text{m s}^{-1}))$		B1
	KE $= \dfrac{1}{2}\times 0.7\times\left(3\sqrt{2}\right)^2$ (J)	Correct for their v at the top, $v \neq 0$	M1
	$= 6.3$ (J)	accept awrt 6.30. CSO	A1
			(3)
7(c)	When P is moving upwards at 6 m s^{-1}	Use suvat to find first time $v = 6$	M1
	$u\sin\theta - gt = 3\sqrt{2}$		A1
	$2g - 3\sqrt{2} - gt = 3\sqrt{2}$	Solve for t	M1
	$t = \dfrac{2g - 6\sqrt{2}}{g} = 1.13....$	Sensitive to premature approximation. Allow 1.14.	A1
	$T = 2 - 1.13 = 0.87$	CAO accept awrt 0.87	A1
			(5)
	Alternative		
	$6\sin 45 = 0 + gt$	find time from top to A:	M1A1
	$T = 2t = \dfrac{\frac{12\sqrt{2}}{2}}{g} = 0.87$	Correct strategy Correct unsimplified	M1 A1 A1
			(5)

Question	Scheme		Marks
7(c) *continued*	**Alternative**		
	: $u\sin\theta = gt$ (their u,θ)	Time to top	M1
	$t = 1.567....$		A1
	$T = 2(2-1.567...)$		M1A1
	$= 0.87$		A1
			(5)
	Alternative		
	Vertical speed at A = - (vertical speed at B) = $= \sqrt{36-(3\sqrt{2})^2} = 3\sqrt{2}$	Or use the 45° angle	M1 A1
	Use $v = u + at$ for $A \to B$	Correct use for their values	M1
	$-3\sqrt{2} = 3\sqrt{2} - gT$		A1
	$T = 0.87$		A1
	See below for alt 7d		**(5)**
	Alternative 7d		
	$v^2 = (3\sqrt{2})^2 + (u\sin\theta - gt)^2 \le 36$	Form expression for v^2. Inequality not needed at this stage	M1
		Correct inequality for v^2.	A1
	$-\sqrt{18} \le u\sin\theta - gt \le \sqrt{18}$		M1
	$\dfrac{u\sin\theta - \sqrt{18}}{g} \le t \le \dfrac{u\sin\theta + \sqrt{18}}{g}$		A1
	$T = \dfrac{u\sin\theta + \sqrt{18}}{g} - \dfrac{u\sin\theta - \sqrt{18}}{g} = \dfrac{2\sqrt{18}}{g} = 0.866$		A1
			(5)
			(14 marks)

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M3

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WME13/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

1.

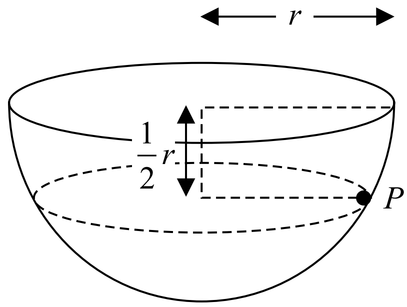

Figure 1

A hemispherical bowl, of internal radius r, is fixed with its circular rim upwards and horizontal. A particle P of mass m moves on the smooth inner surface of the bowl. The particle moves with constant angular speed in a horizontal circle. The centre of the circle is at a distance $\frac{1}{2}r$ vertically below the centre of the bowl, as shown in Figure 1.

The time taken by P to complete one revolution of its circular path is T.

Show that $T = \pi\sqrt{\frac{2r}{g}}$.

(8)

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Question 1 continued

Q1

(Total for Question 1 is 8 marks)

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2. A spacecraft S of mass m moves in a straight line towards the centre of the Earth. The Earth is modelled as a sphere of radius R and S is modelled as a particle. When S is at a distance x, $x \geqslant R$, from the centre of the Earth, the force exerted by the Earth on S is directed towards the centre of the Earth. The force has magnitude $\dfrac{K}{x^2}$, where K is a constant.

(a) Show that $K = mgR^2$

(2)

When S is at a distance $3R$ from the centre of the Earth, the speed of S is V. Assuming that air resistance can be ignored,

(b) find, in terms of g, R and V, the speed of S as it hits the surface of the Earth.

(7)

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Question 2 continued

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Question 2 continued

Q2

(Total for Question 2 is 9 marks)

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3. At time $t = 0$, a particle P is at the origin O, moving with speed 8 m s^{-1} in the positive x direction. At time t seconds, $t \geqslant 0$, the acceleration of P has magnitude $2(t+4)^{-\frac{1}{2}}$ m s^{-2} and is directed towards O.

(a) Show that, at time t seconds, the velocity of P is $16 - 4(t+4)^{\frac{1}{2}}$ m s^{-1}

(5)

(b) Find the distance of P from O when P comes to instantaneous rest.

(7)

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Question 3 continued

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Question 3 continued

Q3

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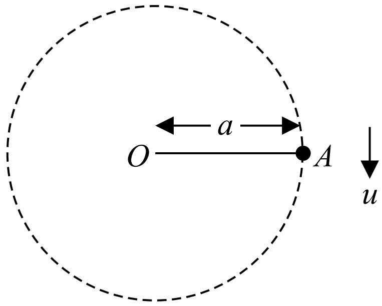

Figure 2

A particle of mass $3m$ is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held at the point A, where OA is horizontal and $OA = a$. The particle is projected vertically downwards from A with speed u, as shown in Figure 2. The particle moves in complete vertical circles.

(a) Show that $u^2 \geqslant 3ag$.

(7)

Given that the greatest tension in the string is three times the least tension in the string,

(b) show that $u^2 = 6ag$.

(5)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total for Question 4 is 12 marks)

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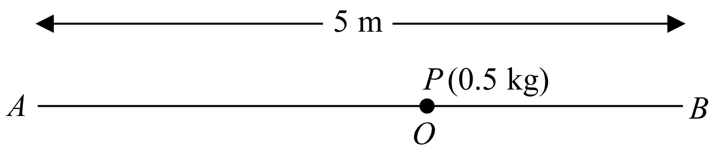

Figure 3

Two fixed points A and B are 5 m apart on a smooth horizontal floor. A particle P of mass 0.5 kg is attached to one end of a light elastic string, of natural length 2 m and modulus of elasticity 20 N. The other end of the string is attached to A. A second light elastic string, of natural length 1.2 m and modulus of elasticity 15 N, has one end attached to P and the other end attached to B.

Initially P rests in equilibrium at the point O, as shown in Figure 3.

(a) Show that $AO = 3$ m.

(4)

The particle is now pulled towards A and released from rest at the point C, where ACB is a straight line and $OC = 1$ m.

(b) Show that, while both strings are taut, P moves with simple harmonic motion.

(4)

(c) Find the speed of P at the instant when the string PB becomes slack.

(4)

The particle first comes to instantaneous rest at the point D.

(d) Find the distance DB.

(5)

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Question 5 continued

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Question 5 continued

Q5

(Total for Question 5 is 17 marks)

6.

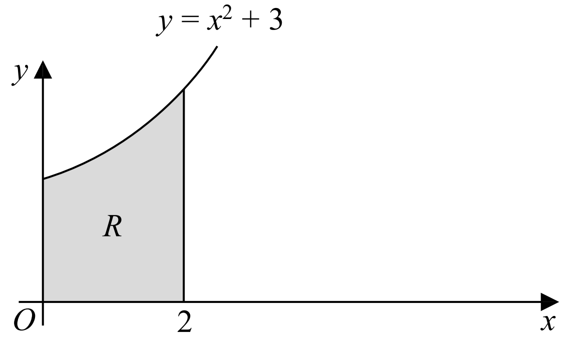

Figure 4

The shaded region R is bounded by part of the curve with equation $y = x^2 + 3$, the x-axis, the y-axis and the line with equation $x = 2$, as shown in Figure 4. The unit of length on each axis is one centimetre. The region R is rotated through 2π radians about the x-axis to form a uniform solid S.
Using algebraic integration,

(a) show that the volume of S is $\dfrac{202}{5}\pi\,\text{cm}^3$,

(4)

(b) show that, to 2 decimal places, the centre of mass of S is 1.30 cm from O.

(5)

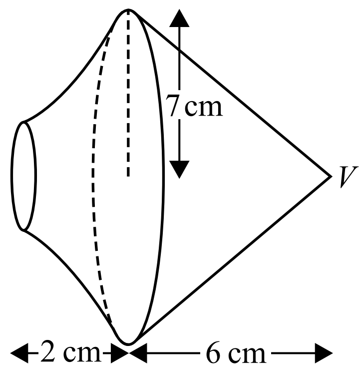

Figure 5

A uniform right circular solid cone, of base radius 7 cm and height 6 cm, is joined to S to form a solid T. The base of the cone coincides with the larger plane face of S, as shown in Figure 5. The vertex of the cone is V.
The mass per unit volume of S is twice the mass per unit volume of the cone.

(c) Find the distance from V to the centre of mass of T.

(5)

The point A lies on the circumference of the base of the cone. The solid T is suspended from A and hangs freely in equilibrium.

(d) Find the size of the angle between VA and the vertical.

(3)

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Question 6 continued

Q6

(Total for Question 6 is 17 marks)

TOTAL FOR PAPER IS 75 MARKS

Mechanics M3 Mark scheme

Question	Scheme	Marks
1	(30° or θ for the first 3 lines)	
	$R\sin 30° = mg$	M1 A1
	$R\cos 30° = m\left(r\cos 30°\right)\omega^2$	M1 A1 A1
	$\omega^2 = \dfrac{R}{mr} = \dfrac{g}{r\sin 30}$	DM1
	$\omega = \sqrt{\dfrac{2g}{r}}$	A1
	Time = $\dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{r}{2g}} = \pi\sqrt{\dfrac{2r}{g}}$ *	A1 cso
		(8)
	Alternative:	
	Resolve perpendicular to the reaction:	
	$mg\cos 30 = m \times rad \times \omega^2 \cos 60$	M2 A1 (LHS) A1 (RHS)
	$= mr\cos 30\omega^2 \cos 60$	A1
	Obtain ω	M1 A1
	Correct time	A1
		(8)
		(8 marks)

Notes:

M1: Resolving vertically 30° or θ

A1: Correct equation 30° or θ

M1: Attempting an equation of motion along the radius, acceleration in either form 30° or θ
Allow with r for radius.

A1: LHS correct 30° or θ

A1: RHS correct, 30° or θ but not r for radius.

DM1: Obtaining an expression for ω^2 or for v^2 **and** the length of the path 30° or θ Dependent on both previous M marks.

A1: Correct expression for ω Must have the numerical value for the trig function now.

A1cso: Deducing the GIVEN answer.

Question	Scheme	Marks
2(a)	$F=\dfrac{K}{x^2}$	
	$x=R\Rightarrow F=mg \qquad \therefore mg=\dfrac{K}{R^2}$	M1
	$K=mgR^2$ *	A1
		(2)
(b)	$\dfrac{mgR^2}{x^2}=-mv\dfrac{\mathrm{d}v}{\mathrm{d}x}$	M1
	$g\int\dfrac{R^2}{x^2}\mathrm{d}x=-\int v\,\mathrm{d}v$	
	$-g\dfrac{R^2}{x}=-\dfrac{1}{2}v^2 \quad (+c)$	dM1 A1ft
	$x=3R,\ v=V\Rightarrow -g\dfrac{R^2}{3R}=-\dfrac{1}{2}V^2+c$	M1
	$c=-\dfrac{Rg}{3}+\dfrac{1}{2}V^2$	A1
	$x=R\Rightarrow\dfrac{1}{2}v^2=-\dfrac{Rg}{3}+\dfrac{1}{2}V^2+g\dfrac{R^2}{R}$	M1
	$v^2=V^2+\dfrac{4Rg}{3}$	
	$v=\sqrt{V^2+\dfrac{4Rg}{3}}$	A1 cso
		(7)
		(9 marks)

Notes:

(a)

M1: Setting $F=mg$ **and** $x=R$

A1: Deducing the GIVEN answer

(b)

M1: Attempting an equation of motion with acceleration in the form $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$. The minus sign may be missing.

dM1: Attempting the integration.

A1ft: Correct integration, follow through on a missing minus sign from line 1, constant of integration may be missing.

M1: Substituting $x=3R, v=V$ to obtain an equation for c

A1: Correct expression for c.

M1: Substituting $x=R$ and their expression for c.

A1: Correct expression for v, any equivalent form.

Question	Scheme	Marks
3(a)	$\frac{\mathrm{d}v}{\mathrm{d}t} = -2(t+4)^{-\frac{1}{2}}$	M1
	$v = -\int 2(t+4)^{-\frac{1}{2}}\,\mathrm{d}t$	
	$v = -4(t+4)^{\frac{1}{2}} \quad (+c)$	dM1 A1
	$t = 0, v = 8 \Rightarrow c = 16$	M1
	$v = 16 - 4(t+4)^{\frac{1}{2}}$ $(\mathrm{m\ s^{-1}})$ *	A1 **cso**
		(5)
(b)	$v = 0 \quad 16 = 4(t+4)^{\frac{1}{2}}$	M1
	$16 = t+4 \quad t = 12$	A1
	$x = 4\int\left(4-(t+4)^{\frac{1}{2}}\right)\mathrm{d}t$	
	$x = 4\left(4t - \frac{2}{3}(t+4)^{\frac{3}{2}}\right) \quad (+d)$	M1 A1
	$t = 0,\ x = 0 \quad d = 4\times\frac{2}{3}\times 4^{\frac{3}{2}} = \frac{64}{3}$ oe	A1
	$t = 12 \quad x = 4\left(4\times 12 - \frac{2}{3}\times 16^{\frac{3}{2}}\right) + \frac{64}{3} = 42\frac{2}{3}$ (m) oe eg 43 or better	dM1 A1
		(7)
		(12 marks)

Notes:

(a)

M1: Attempting an expression for the acceleration in the form $\frac{\mathrm{d}v}{\mathrm{d}t}$; minus may be omitted.

DM1: Attempting the integration

A1: Correct integration, constant of integration may be omitted (no ft)

M1: Using the initial conditions to obtain a value for the constant of integration

A1: **cso.** Substitute the value of c and obtain the final GIVEN answer

(b)

M1: Setting the **given** expression for v equal to 0

A1: Solving to get $t = 12$

M1: Setting $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ and attempting the integration wrt t. At least one term must clearly be integrated.

A1: Correct integration, constant may be omitted.

Question 3 notes *continued*	
M1:	Substituting $t=0,\ x=0$ and obtaining the correct value of d. Any equivalent number, inc decimals.
dM1:	Substituting their value for t and obtaining a value for the required distance. Dependent on the second M mark.
A1:	Correct final answer, any equivalent form.

Question	Scheme	Marks
4(a)	Energy to top: $\frac{1}{2}\times 3m\times u^2-\frac{1}{2}\times 3mv^2=3mga$	M1 A1
	NL2 at top: $T+3mg=3m\frac{v^2}{a}$	M1 A1
	$T=3m\frac{u^2}{a}-6mg-3mg$	dM1
	$T\geqslant 0\ \Rightarrow\frac{u^2}{a}\geqslant 3g$	M1
	$u^2\geqslant 3ag$	A1 cso
		(7)
(b)	Tension at bottom:	
	$\frac{1}{2}\times 3m\times V^2-\frac{1}{2}\times 3mu^2=3mga$	M1
	$T_{\max}-3mg=3m\frac{V^2}{a}$	M1
	$T_{\max}=3mg+6mg+3m\frac{u^2}{a}$	A1
	$T_{\min}=3m\frac{u^2}{a}-9mg$	
	$9mg+3m\frac{u^2}{a}=3\left(3m\frac{u^2}{a}-9mg\right)$	dM1
	$u^2=6ag$ *	A1 cso
		(5)
		(12 marks)

Notes:

(a)

M1: Attempting an energy equation, can be to a general point for this mark. Mass can be missing but use of $v^2=u^2+2as$ scores M0

A1: Correct equation from A to the top.

M1: Attempting an equation of motion along the radius at the top, acceleration in either form.

A1: Correct equation, acceleration in form $\frac{v^2}{r}$

dM1: Eliminate v^2 to obtain an expression for T dependent on both previous M marks.

M1: Use $T\geqslant 0$ at top to obtain an inequality connecting a, g and u

A1: Re-arrange to obtain the GIVEN answer.

Question 4 notes *continued*	
(b)	
M1:	Attempting an energy equation to the bottom, maybe from A or from the top.
M1:	Attempting an equation of motion along the radius at the bottom.
A1:	Correct expression for the max tension.
dM1:	Forming an equation connecting *their* tension at the top with *their* tension at the bottom. If the 3 is multiplying the wrong tension this mark can still be gained. Dependent on both previous M marks.
A1:	**cso.** Obtaining the GIVEN answer.

Question	Scheme	Marks
5(a)	$T=\dfrac{20e}{2}=\dfrac{15(1.8-e)}{1.2}$	M1A1
	$10e\times 1.2=15(1.8-e)$	
	$e=1$	A1
	$AO=3\text{m}$ *	A1cso
		(4)
(b)	$0.5\ddot{x}=\dfrac{20(1-x)}{2}-\dfrac{15(0.8+x)}{1.2}$	M1 A1 A1
	$\ddot{x}=-45x \quad \therefore \text{SHM}$	A1 cso
		(4)
(c)	String becomes slack when $x=(-)0.8$ (allow wo sign due to symmetry)	B1
	$v^2=\omega^2\left(a^2-x^2\right)$	
	$v^2=45\left(1-0.8^2\right) \quad (=16.2)$	M1 A1 ft
	$v=4.024...$ m s^{-1} (4.0 or better)	A1ft
		(4)
(d)	$\dfrac{1}{2}\times\dfrac{20y^2}{2}-\dfrac{1}{2}\times\dfrac{20\times 1.8^2}{2}=\dfrac{1}{2}\times 0.5\times 16.2$ ft on v	M1 A1 A1 ft
	$20y^2-64.8=16.2$	
	$y^2=4.05 \quad y=2.012....$	A1
	Distance $DB=\lvert 5-4.012...\rvert=0.988...\text{m}$ (accept 0.99 or better)	A1ft
	Alternative	
	$0.5\text{a}=-10(1.8+x)$	
	$v\dfrac{\text{d}v}{\text{d}x}=-36-10x$	
	$\int v\,\text{d}v=-\int(36+10x)\,\text{d}x$	
	$\dfrac{v^2}{2}=-36x+5x^2+c$	M1 A1
	$x=0,\ v=\dfrac{9\sqrt{5}}{5}\ \therefore\ c=8.1$	A1
	Then $v=0$ etc	M1 A1
		(5)
		(17 marks)

Question 5 *continued*

Notes:

(a)

M1: Attempting to obtain and equate the tensions in the two parts of the string.

A1: Correct equation, extension in AP or BP can be used or use OA as the unknown.

A1: Obtaining the correct extension in either string (ext in BP =0.8 m) or another useful distance.

A1: **cso.** Obtaining the correct GIVEN answer.

(b)

M1: Forming an equation of motion at a general point. There must be a difference of tensions, both with the variable. May have m instead of 0.5 Accel can be a.

A1 A1: Deduct 1 for each error, m or 0.5 allowed, acceleration to be $\ddot{x}$ now.

A1: **cso** Correct equation in the required form, with a concluding statement; m or 0.5 allowed.

Question 5 notes *continued*

(c)

B1: For $x = \pm 0.8$ Need not be shown explicitly.

M1: Using $v^2 = \omega^2\left(a^2 - x^2\right)$ with *their* (numerical) ω and their x

A1ft: Equation with correct numbers ft their ω

A1ft: Correct value for v 2sf or better or exact.

(d)

M1: Attempting an energy equation with 2 EPE terms and a KE term.

A1: 2 correct terms may have $(1.8 + x)$ instead of y.

A1ft: Completely correct equation, follow through their v from (c)

A1: Correct value for distance travelled after PB became slack. $x = 0.21$

A1ft: Complete to the distance DB. Follow through their distance travelled after PB became slack.

Question	Scheme	Marks
6(a)	$\text{Vol} = \pi\int_0^2 \left(x^2+3\right)^2 \mathrm{d}x$	M1
	$= \pi\int_0^2 \left(x^4+6x^2+9\right)\mathrm{d}x$	
	$= \pi\left[\frac{1}{5}x^5+2x^3+9x\right]_0^2$	dM1 A1
	$= \frac{202}{5}\pi \text{ cm}^3$ *	A1
		(4)
(b)	$\pi\int_0^2 x\left(x^2+3\right)^2 \mathrm{d}x = \pi\int_0^2\left(x^5+6x^3+9x\right)\mathrm{d}x$	M1
	$= \pi\left[\frac{1}{6}x^6+\frac{3}{2}x^4+\frac{9}{2}x^2\right]_0^2$	A1
	$= \frac{158}{3}\pi$ (Or by chain rule or substitution)	A1
	C of m $= \frac{158}{3}\times\frac{5}{202}, \; = 1.3036... = 1.30$ cm	M1 A1
		(5)
(c)	Mass ratio $\quad 2\times\frac{202}{5}\pi \quad\quad \frac{1}{3}\pi\times7^2\times6 \quad\quad \left(\frac{404}{5}+98\right)\pi$	B1
	Dist from V $\quad 6.7 \quad\quad 4.5 \quad\quad \bar{x}$	B1
	$\frac{404}{5}\times6.7+98\times4.5 = \left(\frac{404}{5}+98\right)\bar{x}$	M1 A1 ft
	$\bar{x} = \dfrac{\frac{404}{5}\times6.7+98\times4.5}{\left(\frac{404}{5}+98\right)} = 5.494.... = 5.5$ cm $\quad$ Accept 5.49 or better	A1
		(5)
(d)	$\tan\theta = \frac{6-\bar{x}}{7} = \frac{0.5058...}{7}$	M1
	$\alpha = \tan^{-1}\left(\frac{6}{7}\right)-\tan^{-1}\left(\frac{0.5058...}{7}\right) = 36.468...° = 36°$ $\quad$ or better	M1 A1
		(3)
		(17 marks)

Notes:

(a)

M1: Using $\pi\int y^2\mathrm{d}x$ with the equation of the curve, no limits needed

Question 6 notes *continued*	
dM1:	Integrating their expression for the volume.
A1:	Correct integration inc limits now.
A1:	Substituting the limits to obtain the GIVEN answer.
(b)	
M1:	Using $(\pi)\int xy^2\mathrm{d}x$ with the equation of the curve, no limits needed, π can be omitted.
A1:	Correct integration, including limits; no substitution needed for this mark.
A1:	Correct substitution of limits.
M1:	Use of $\dfrac{\pi\int xy^2\mathrm{d}x}{\pi\int y^2\mathrm{d}x}$ with their $\pi\int xy^2\mathrm{d}x$. π must be seen in both numerator and denominator or in neither.
A1:	**cso.** Correct answer. Must be 1.30
(c)	
B1:	Correct mass ratio.
B1:	Correct distances, from V or any other point, provided consistent.
M1:	Attempting a moments equation.
A1ft:	Correct equation, follow through their distances and mass ratio.
A1:	Correct distance from V
(d)	
M1:	Attempting the tan of an appropriate angle, numbers either way up.
M1:	Attempting to obtain the required angle.
A1:	Correct final answer 2sf or more.

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S1

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WST11/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. The percentage oil content, p, and the weight, w milligrams, of each of 10 randomly selected sunflower seeds were recorded. These data are summarised below.

$$\sum w^2 = 41252 \qquad \sum wp = 27\,557.8 \qquad \sum w = 640 \qquad \sum p = 431 \qquad S_{pp} = 2.72$$

(a) Find the value of S_{ww} and the value of S_{wp}

(3)

(b) Calculate the product moment correlation coefficient between p and w

(2)

(c) Give an interpretation of your product moment correlation coefficient.

(1)

The equation of the regression line of p on w is given in the form $p = a + bw$

(d) Find the equation of the regression line of p on w

(4)

(e) Hence estimate the percentage oil content of a sunflower seed which weighs 60 milligrams.

(2)

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Question 1 continued

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Question 1 continued

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Question 1 continued

Q1

(Total for Question 1 is 12 marks)

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2. The time taken to complete a puzzle, in minutes, is recorded for each person in a club. The times are summarised in a grouped frequency distribution and represented by a histogram.

One of the class intervals has a frequency of 20 and is shown by a bar of width 1.5 cm and height 12 cm on the histogram. The total area under the histogram is 94.5 cm^2

Find the number of people in the club.

(3)

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Question 2 continued

Q2

(Total for Question 2 is 3 marks)

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3. The discrete random variable X has probability distribution

$$P(X = x) = \frac{1}{5} \qquad x = 1, 2, 3, 4, 5$$

(a) Write down the name given to this distribution.

(1)

Find

(b) $P(X = 4)$

(1)

(c) $F(3)$

(1)

(d) $P(3X - 3 > X + 4)$

(2)

(e) Write down $E(X)$

(1)

(f) Find $E(X^2)$

(2)

(g) Hence find $\text{Var}(X)$

(2)

Given that $E(aX - 3) = 11.4$

(h) find $\text{Var}(aX - 3)$

(4)

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Question 3 continued

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Question 3 continued

Q3

(Total for Question 3 is 14 marks)

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4. A researcher recorded the time, t minutes, spent using a mobile phone during a particular afternoon, for each child in a club.

The researcher coded the data using $v = \dfrac{t-5}{10}$ and the results are summarised in the table below.

Coded Time (v)	**Frequency (f)**	**Coded Time Midpoint (m)**
$0 \leqslant v < 5$	20	2.5
$5 \leqslant v < 10$	24	a
$10 \leqslant v < 15$	16	12.5
$15 \leqslant v < 20$	14	17.5
$20 \leqslant v < 30$	6	b

(You may use $\sum fm = 825$ and $\sum fm^2 = 12\,012.5$)

(a) Write down the value of a and the value of b.
(1)

(b) Calculate an estimate of the mean of v.
(1)

(c) Calculate an estimate of the standard deviation of v.
(2)

(d) Use linear interpolation to estimate the median of v.
(2)

(e) Hence describe the skewness of the distribution. Give a reason for your answer.
(2)

(f) Calculate estimates of the mean and the standard deviation of the time spent using a mobile phone during the afternoon by the children in this club.
(4)

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Question 4 continued

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Question 4 continued

Q4

(Total for Question 4 is 12 marks)

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5. A biased tetrahedral die has faces numbered 0, 1, 2 and 3. The die is rolled and the number face down on the die, X, is recorded. The probability distribution of X is

x	0	1	2	3
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$

If $X = 3$ then the final score is 3
If $X \neq 3$ then the die is rolled again and the final score is the sum of the two numbers.

The random variable T is the final score.

(a) Find $P(T = 2)$

(2)

(b) Find $P(T = 3)$

(3)

(c) Given that the die is rolled twice, find the probability that the final score is 3

(3)

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Question 5 continued

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Question 5 continued

Q5

(Total for Question 5 is 8 marks)

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6. Three events A, B and C are such that

$$\mathrm{P}(A) = \frac{2}{5} \qquad \mathrm{P}(C) = \frac{1}{2} \qquad \mathrm{P}(A \cup B) = \frac{5}{8}$$

Given that A and C are mutually exclusive find

(a) $\mathrm{P}(A \cup C)$

(1)

Given that A and B are independent

(b) show that $\mathrm{P}(B) = \dfrac{3}{8}$

(4)

(c) Find $\mathrm{P}(A \mid B)$

(1)

Given that $\mathrm{P}(C' \cap B') = 0.3$

(d) draw a Venn diagram to represent the events A, B and C

(5)

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Question 6 continued

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Question 6 continued

Q6

(Total for Question 6 is 11 marks)

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7. A machine fills bottles with water. The volume of water delivered by the machine to a bottle is X ml where $X \sim \mathrm{N}(\mu, \sigma^2)$

One of these bottles of water is selected at random.

Given that $\mu = 503$ and $\sigma = 1.6$

(a) find

(i) $\mathrm{P}(X > 505)$
(ii) $\mathrm{P}(501 < X < 505)$

(5)

(b) Find w such that $\mathrm{P}(1006 - w < X < w) = 0.9426$

(3)

Following adjustments to the machine, the volume of water delivered by the machine to a bottle is such that $\mu = 503$ and $\sigma = q$

Given that $\mathrm{P}(X < r) = 0.01$ and $\mathrm{P}(X > r + 6) = 0.05$

(c) find the value of r and the value of q

(7)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

Statistics S1 Mark scheme

Question	Scheme	Marks
1(a)	$S_{ww} = 41252 - \frac{640^2}{10} =$ **292**	M1A1
	$S_{wp} = 27557.8 - \frac{640 \times 431}{10} =$ **–26.2**	A1
		(3)
(b)	$r = \frac{-26.2}{\sqrt{292 \times 2.72}}$	M1
	$= -0.9297$ awrt **–0.930**	A1
		(2)
(c)	As weight increases the percentage of oil content decreases o.e.	B1
		(1)
(d)	$b = \frac{-26.2}{292} = -0.0897...$ awrt **–0.09**	M1 A1
	$a = \frac{431}{10} - \left(\frac{-26.2}{292}\right) \times \left(\frac{640}{10}\right) = 48.842\ldots$	M1
	p* = 48.8 – 0.0897*w	A1
		(4)
(e)	$p = 48.8 - 0.0897 \times 60$	M1
	= 43.4/43.5 awrt **43.4/43.5**	A1
		(2)
		(12 marks)

Notes:

(a)
M1: for a correct expression for S_{ww} or S_{wp} (may be implied by one correct answer)
1st A1: for either $S_{ww} = 292$ or $S_{wp} = -26.2$
2nd A1: for **both** $S_{ww} = 292$ and $S_{wp} = -26.2$

(b)
M1: for a correct expression (Allow ft of their S_{ww} or S_{wp} provided $S_{ww} \neq 41252$ and $S_{wp} \neq 27557.8$). Condone missing "–"
A1: for awrt –0.930 (Condone –0.93 for M1A1 if correct expression is seen) (Answer only awrt –0.930 scores 2/2 but answer only –0.93 is M1A0)

(c)
B1: For a correct contextual description of negative correlation which must include weight and oil (but *w* increases as *p* decreases is not sufficient)

(d)
1st M1: for a correct expression for *b* (Allow ft)
1st A1: for awrt –0.09
2nd M1: for a correct method for *a* ft their value of *b* (Allow $a = 43.1 + b \times 64$)
2nd A1: for a correct equation for *p* and *w* with *a* = awrt 48.8 and *b* = awrt –0.0897 No fractions. Equation in *x* and *y* is A0

(e)
M1: substituting *w* = 60 into their equation
A1: awrt 43.4 or 43.5 (Answer only scores 2/2)

Question	Scheme	Marks
2	$1.5\times12 = 18$ 20 people represented by 18 (cm^2) or 1 person is represented by 0.9 (cm^2)	M1
	$x = \dfrac{20\times94.5}{18}$ oe = 105 (people)	M1 A1 cao
		(3 marks)

Notes:

M1: For an attempt to relate area to frequency (e.g. $\frac{20}{18}$ or $\frac{18}{20}$ seen)

M1: For a correct expression/equation for total frequency e.g. $\frac{18}{20} = \frac{94.5}{x}$

A1: For 105 cao

Question	Scheme	Marks
3(a)	(Discrete) **Uniform**	B1
		(1)
(b)	$P(X = 4) = \underline{\frac{1}{5}}$ oe	B1
		(1)
(c)	$F(3) = \underline{\frac{3}{5}}$ oe	B1
		(1)
(d)	$P(3X - 3 > X + 4) = P(X > 3.5)$	M1
	$\underline{= \frac{2}{5}}$ oe	A1
		(2)
(e)	$E(X) = \underline{\mathbf{3}}$	
		B1
		(1)
(f)	$E(X^2) = \frac{1}{5}\left(1^2 + 2^2 + 3^2 + 4^2 + 5^2\right)$	M1
	$= \underline{\mathbf{11}}$	A1
		(2)
(g)	$\text{Var}(X) = 11 - 3^2$ or $\frac{(5+1)(5-1)}{12}$	M1
	$= \underline{\mathbf{2}}$	A1
		(2)
(h)	$11.4 = aE(X) - 3$ **or** $11.4 = 3a - 3$	M1
	$a = 4.8$	A1
	$\text{Var}(4.8X - 3) = \text{'}4.8\text{'}^2 \times \text{'}2\text{'}$	M1
	$= 46.08$ awrt $\underline{\mathbf{46.1}}$	A1
		(4)
		(14 marks)

Question 3 *continued*
Notes:
(a) **B1:** For uniform.
(d) **M1:** For identifying the correct probabilities i.e. $P(X > 3.5)$ <u>or</u> $P(X = 4) + P(X = 5)$
(f) **M1:** For a correct expression.
(g) **M1:** For either 'their (f)' – 'their (e)'^{2} <u>or</u> for a correct expression $\frac{(5+1)(5-1)}{12}$
(h) **1st M1:** For setting up a correct linear equation using $aE(X) - 3 = 11.4$ **1st A1:** May be implied by a correct answer. **2nd M1:** For $\text{"their } a^2\text{"} \times \text{"their Var}(X)\text{"}$ (must see values substituted) (may be implied by a correct answer or correct ft answer) NB: 'their Var(X)' < 0 is M0 here.

Question	Scheme	Marks
4(a)	7.5 <u>and</u> 25	B1
		(1)
(b)	Mean = 10.3125 awrt **<u>10.3</u>**	B1
		(1)
(c)	$\sigma = \sqrt{\frac{120125}{80} - '10.3125'^2}$	M1
	= 6.6188.. (s = 6.6605…) awrt **<u>6.62</u>**	A1
		(2)
(d)	Median = $\{5\} + \frac{20}{24} \times 5$ or $\{10\} - \frac{4}{24} \times 5$	M1
	= 9.16666 awrt **<u>9.17</u>**	A1
		(2)
(e)	Mean > median $\therefore$ positive skew	M1A1
		(2)
(f)	$t = 10v + 5$	
	Mean = $10 \times 10.3125 + 5$	M1
	=108.125 awrt **<u>108</u>**	A1
	$\sigma = 10 \times 6.6188$	M1
	= 66.188.. (66.605 from s) awrt **<u>66.2</u>**	A1
		(4)
		(12 marks)

Notes:

(a)
B1: Both values correct (may be seen in table)

(b)
B1: For awrt 10.3 (Do not allow improper fractions).

(c)
M1: For a correct expression including the square root (allow ft from their mean)
A1: For awrt 6.62 (Allow s = awrt 6.66)

(d)
M1: For a correct fraction: $\frac{20}{24} \times 5$ <u>or</u> if using n + 1 for $\frac{20.5}{24} \times 5$ may be scored from working down $-\frac{4}{24} \times 5$
A1: For awrt 9.17 or (if using n + 1) for awrt 9.27

Question 4 notes *continued*
(e) **M1:** For a correct comparison of 'their b' and 'their d' (must have an answer to both (b) and (d)) Comparison may be part of bigger expression e.g. 3(mean – median)/s.d. Allow use of $Q_3 - Q_2 > Q_2 - Q_1$ only if $Q_1 = 5$ and $Q_3 = 15$ are both seen **A1:** For positive skew (which must follow from their values)
(f) **M1:** **(1st M1)** For $10\times\text{"their mean"}+5$ **M1:** **(2nd M1)** or $10\times\text{"their sd"}$ Use of decoded data to find mean must be fully correct, i.e. 8650/80 = awrt 108 (M1A1) Use of decoded data to find s.d. must be fully correct, i.e. $\sqrt{\frac{1285750}{80}-\left(\frac{8650}{80}\right)^2}$ =awrt 66.2 (M1A1)

Question	Scheme	Marks
5(a)	$\mathrm{P}(T=2)=3\times\frac{1}{6}\times\frac{1}{6}=\frac{1}{12}$ oe	M1 A1
		(2)
(b)	$\mathrm{P}(T=3)=[\mathrm{P}(0,3)+\mathrm{P}(1,2)+\mathrm{P}(2,1)]+\mathrm{P}(3)$	
	$=\left(\frac{1}{6}\times\frac{1}{2}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{1}{6}\times\frac{1}{6}\right)+\frac{1}{2}$	M1 M1
	$=\frac{23}{36}$ oe	A1
		(3)
(c)	$\mathrm{P}(T=3 \mid \text{rolled twice})=\frac{\mathrm{P}((T=3)\cap\text{die rolled twice})}{\mathrm{P}(\text{die rolled twice})}$	M1
	$=\frac{\frac{5}{36}}{\frac{1}{2}}$	M1
	$=\frac{5}{18}$ oe	A1
		(3)
		(8 marks)

Notes:

Correct answer only in (a), (b) or (c) scores full marks for that part.
Methods leading to answers > 1 score 0 marks

(a)

M1: For a correct expression.

A1: Allow exact equivalent ($\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$ is M0A0).

(b)

M1: For $\frac{1}{2}$ + at least one correct product.

M1: For fully correct expression.

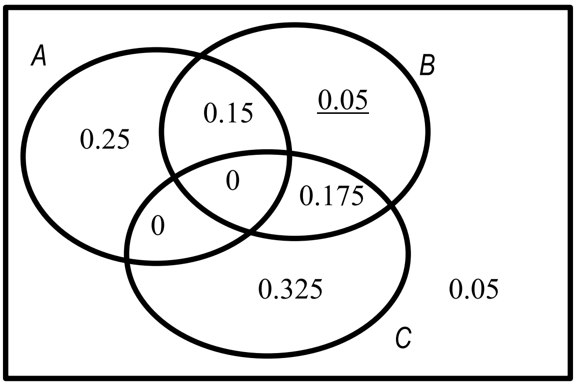
A1: Allow exact equivalent.

(c)

M1: For correct conditional probability ratio (this mark may be implied by 2nd M1) **but** going on to assume independence [using numerator $\mathrm{P}(T=3)\times\mathrm{P}(\text{rolled twice})$] is M0M0A0.

M1: For a correct numerical ratio of probabilities (allow ft of (their (b) – $\frac{1}{2}$) as numerator).

A1: Allow exact equivalent.

Question	Scheme	Marks
6(a)	$[\text{P}(A \cup C) =] \underline{\frac{9}{10}}$ oe	B1
		(1)
(b)	$\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A) \times \text{P}(B)$	M1
	$\frac{5}{8} = \frac{2}{5} + \text{P}(B) - \frac{2}{5}\text{P}(B)$	M1 A1
	$\text{P}(B) = \frac{3}{8}$ *	A1cso
		(4)
(c)	$[\text{P}(A \mid B) = \text{P}(A) =] \underline{\frac{2}{5}}$ oe	B1
		(1)
(d)	 Diagram	B1
	0.15 <u>and</u> 0.25	M1
	0.05 <u>and</u> <u>0.05</u>	M1
	0.175 <u>and</u> 0.325	M1 A1
		(5)
		(11 marks)

Notes:

(b)

M1: For use of $\text{P}(A \cup B) = \text{P}(A) + P(B) - \text{P}(A \cap B)$

M1: For use of $\text{P}(A \cap B) = \text{P}(A) \times \text{P}(B)$ (But just seeing $\frac{2}{5} \times \frac{3}{8} = \frac{3}{20}$ on its own is M0M0)

A1: A correct equation

A1: (No wrong working seen dependent on all previous marks)
(allow a full verification method, however, substitution of P(B) = 3/8 into only one P(B) to find the other P(B) (e.g. using 3/20 to find 3/8) can score M1M0A0A0)

Question 6 notes *continued*

(d)

B1: 3 circles intersecting, see diagram above, (at least 2 labelled) with the two zeros showing *A* does not intersect *C* (Do not allow blank spaces for the two zeros)

<u>**or**</u> 3 circles, see diagram below, (at least 2 labelled) where *B* intersects *A* and *C* but *A* and *C* do not intersect.

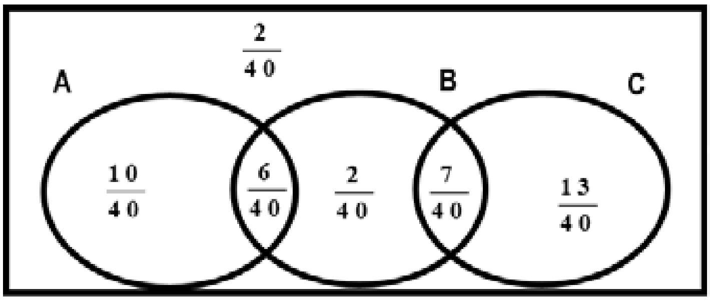
M1: 0.15 placed in $(A \cap B \cap C')$ <u>and</u> 0.25 placed in $(A \cap B' \cap C')$

M1: 0.3 – 'their 0.25' <u>and</u> 1 – ('their 0.15' + 'their 0.25' + 'their 0.05' + $\frac{1}{2}$)

M1: $\frac{3}{8}-$ ("their 0.15" + "their <u>0.05</u>"), i.e. $\mathrm{P}(B)=\frac{3}{8}$ <u>and</u> $\frac{1}{2}-$ "their 0.175", i.e. $\mathrm{P}(C)=\frac{1}{2}$

For the 3rd M mark, blank regions inside P(*B*) and P(*C*) are not treated as 0s and score M0

A1: fully correct with box



Question	Scheme	Marks
7(a)(i)	$\text{P}(X > 505) = \text{P}\left(Z > \dfrac{505-503}{1.6}\right)$	M1
	$= 1 - \text{P}(Z < 1.25) \quad = 1 - 0.8944$	M1
	$= 0.1056$ awrt **<u>0.106</u>**	A1
		(3)
(ii)	$\text{P}(501 < X < 505) = 1 - 2\times 0.1056$ or $0.8944 - 0.1056$	M1
	$= 0.7888$ awrt **<u>0.789</u>**	A1
		(2)
(b)	$\text{P}(X < w) = 0.9713$ or $\text{P}(X > w) = 0.0287$ (may be implied by $z = \pm 1.9$)	M1
	$\dfrac{w-503}{1.6} = 1.9$ or $\dfrac{(1006-w)-503}{1.6} = -1.9$	M1
	$w = 506.04\ldots$ awrt **<u>506</u>**	A1
		(3)
(c)	$\dfrac{r-503}{q} = -2.3263$	M1A1
	$\dfrac{r+6-503}{q} = 1.6449$	M1A1
	$1.6449q - 6 = -2.3263q$	ddM1
	$q = 1.51...$ awrt **<u>1.51</u>**	A1
	$r = 499.48\ldots\ldots$ awrt **<u>499</u>**	A1
		(7)
		(15 marks)

Notes:

(a)

(i)

M1: Standardising with 505, 503 and 1.6. May be implied by use of 1.25 (Allow ±)

M1: For $1 - \text{P}(Z < 1.25)$ i.e. a correct method for finding $\text{P}(Z > 1.25)$, e.g. $1 - p$ where $0.5 < p < 0.99$

(ii)

M1: $1 - 2 \times$ their(i)

(b)

M1: For using symmetry to find the area of one tail (may be seen in a diagram)

M1: A single standardisation with 503, 1.6 and w (or $1006 - w$) <u>and</u> set $= \pm z$ value ($1.8 < |z| < 2$)

A1: For awrt 506 which must come from correct working. (**Answer only**: 506 scores 0/3, but 506.0…with no working send to review)

Question 7 notes *continued*
(c) **M1:** $\dfrac{r-503}{q} = z$ value where $\lvert z \rvert > 2$ **A1:** $\dfrac{r-503}{q} =$ awrt -2.3263 (signs must be compatible) **M1:** $\dfrac{r+6-503}{q} = z$ value where $\lvert z \rvert > 1$ **A1:** $\dfrac{r+6-503}{q} =$ awrt 1.6449 (signs must be compatible) **Special Case:** Less than 4dp z-values: use of awrt 2.32/2.33/2.34 **and** awrt 1.64/1.65 could score M1 A0 M1 and then A1 provided both equations have compatible signs. **3rd M1:**(dep on both Ms) attempt to solve simultaneous equations leading to a value for q or r **3rdA1:** Or awrt 1.51 **4thA1:** For awrt 499 (allow 499.5)

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number	Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S2

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WST12/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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S 5 9 7 6 8 A 0 1 2 8

Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. The number of cars caught speeding per day, by a particular camera, has a Poisson distribution with mean 0.8

(a) Find the probability that in a given 4 day period exactly 3 cars will be caught speeding by this camera.

(3)

A car has been caught speeding by this camera.

(b) Find the probability that the period of time that elapses before the next car is caught speeding by this camera is less than 48 hours.

(3)

Given that 4 cars were caught speeding by this camera in a two day period,

(c) find the probability that 1 was caught on the first day and 3 were caught on the second day.

(5)

Each car that is caught speeding by this camera is fined £60

(d) Using a suitable approximation, find the probability that, in 90 days, the total amount of fines issued will be more than £5000

(5)

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Question 1 continued

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Question 1 continued

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Question 1 continued

Q1

(Total for Question 1 is 16 marks)

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2. A continuous random variable X has cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{5}(x-1) & 1 \leqslant x \leqslant 6 \\ 1 & x > 6 \end{cases}$$

(a) Find $\mathrm{P}(X > 4)$ **(2)**

(b) Write down the value of $\mathrm{P}(X \neq 4)$ **(1)**

(c) Find the probability density function of X, specifying it for all values of x **(2)**

(d) Write down the value of $\mathrm{E}(X)$ **(1)**

(e) Find $\mathrm{Var}(X)$ **(2)**

(f) Hence or otherwise find $\mathrm{E}(3X^2 + 1)$ **(3)**

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Question 2 continued

Q2

(Total for Question 2 is 11 marks)

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3. Explain what you understand by

(a) a statistic,

(1)

(b) a sampling distribution.

(1)

A factory stores screws in packets. A small packet contains 100 screws and a large packet contains 200 screws. The factory keeps small and large packets in the ratio 4:3 respectively.

(c) Find the mean and the variance of the number of screws in the packets stored at the factory.

(3)

A random sample of 3 packets is taken from the factory and Y_1, Y_2 and Y_3 denote the number of screws in each of these packets.

(d) List all the possible samples.

(2)

(e) Find the sampling distribution of $\overline{Y}$

(4)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total for Question 3 is 11 marks)

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4. Accidents occur randomly at a crossroads at a rate of 0.5 per month. A researcher records the number of accidents, X, which occur at the crossroads in a year.

(a) Find $P(5 \leqslant X < 7)$

(3)

A new system is introduced at the crossroads. In the first 18 months, 4 accidents occur at the crossroads.

(b) Test, at the 5% level of significance, whether or not there is reason to believe that the new system has led to a reduction in the mean number of accidents per month. State your hypotheses clearly.

(4)

Leave blank

Question 4 continued

Q4

(Total for Question 4 is 7 marks)

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5. The continuous random variable X has probability density function $\mathrm{f}(x)$ given by

$$\mathrm{f}(x) = \begin{cases} k(x^2 + a) & -1 < x \leqslant 2 \\ 3k & 2 < x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k and a are constants.

Given that $\mathrm{E}(X) = \dfrac{17}{12}$

(a) find the value of k and the value of a

(8)

(b) Write down the mode of X

(1)

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Q5

(Total for Question 5 is 9 marks)

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6. The Headteacher of a school claims that 30% of parents do not support a new curriculum. In a survey of 20 randomly selected parents, the number, X, who do not support the new curriculum is recorded.

Assuming that the Headteacher's claim is correct, find

(a) the probability that $X = 5$

(2)

(b) the mean and the standard deviation of X

(3)

The Director of Studies believes that the proportion of parents who do not support the new curriculum is greater than 30%. Given that in the survey of 20 parents 8 do not support the new curriculum,

(c) test, at the 5% level of significance, the Director of Studies' belief. State your hypotheses clearly.

(5)

The teachers believe that the sample in the original survey was biased and claim that only 25% of the parents are in support of the new curriculum. A second random sample, of size $2n$, is taken and exactly half of this sample supports the new curriculum.

A test is carried out at a 10% level of significance of the teachers' belief using this sample of size $2n$

Using the hypotheses H_0: $p = 0.25$ and H_1: $p > 0.25$

(d) find the minimum value of n for which the outcome of the test is that the teachers' belief is rejected.

(3)

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Question 6 continued

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Question 6 continued

Q6

(Total for Question 6 is 13 marks)

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7. A multiple choice examination paper has n questions where $n > 30$
Each question has 5 answers of which only 1 is correct. A pass on the paper is obtained by answering 30 or more questions correctly.

The probability of obtaining a pass by randomly guessing the answer to each question should not exceed 0.0228

Use a normal approximation to work out the greatest number of questions that could be used.

(8)

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Question 7 continued

Leave blank

Question 7 continued

Q7

(Total for Question 7 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

Statistics S2 Mark scheme

Question	Scheme	Marks
1(a)	$X\sim\text{Po}(3.2)$	B1
	$\text{P}(X=3) = \dfrac{\text{e}^{-3.2}3.2^3}{3!}$	M1
	$= 0.2226$ awrt 0.223	A1
		(3)
(b)	$Y\sim\text{Po}(1.6)$	B1
	$\text{P}(Y \geq 1) = 1 - \text{P}(Y=0)$ $= 1 - \text{e}^{-1.6}$	M1
	$= 0.7981$ awrt 0.798	A1
		(3)
(c)	$X\sim\text{Po}(0.8)$	
	$\dfrac{\text{P}(X=1)\times\text{P}(X=3)}{\text{P}(Y=4)} = \dfrac{\left(\text{e}^{-0.8}\times 0.8\right)\times\left(\dfrac{\text{e}^{-0.8}0.8^3}{3!}\right)}{\dfrac{\text{e}^{-1.6}1.6^4}{4!}}$ $= \dfrac{0.3594\times 0.0383}{0.05513}$	M1 M1 M1 A1
	$= 0.25$	A1
		(5)
(d)	$A\sim\text{Po}(72)$ approximated by $\text{N}(72,72)$	B1
	$\dfrac{5000}{60} = 83.33$	M1
	$\text{P}(A \geq 84) = \text{P}\left(Z \geq \dfrac{83.5-72}{\sqrt{72}}\right)$	M1 M1
	$= \text{P}(Z \geq 1.355\ldots)$	
	$= 0.0869$ awrt 0.087/0.088	A1
		(5)
		(16 marks)

Notes:	
(a) **B1:** For writing or using Po(3.2) **M1:** $\dfrac{\text{e}^{-\lambda}\lambda^3}{3!}$	
(b) **B1:** For writing or using Po(1.6) **M1:** $1 - \text{P}(Y=0)$ or $1 - \text{e}^{-\lambda}$	

Question 1 notes *continued*
(c) **M1:** Using Po(0.8) with $X=1$ or $X=3$ (may be implied by 0.359... or 0.0383...) **M1:** $\left(e^{-\lambda}\times\lambda\right)\times\left(\frac{e^{-\lambda}\lambda^{3}}{3!}\right)$ (consistent lambda) awrt 0.0138 implies 1st 2 M marks **M1:** Correct use of conditional probability with denominator $=\frac{e^{-1.6}1.6^{4}}{4!}$ **A1:** Fully correct expression **A1:** 0.25 (allow awrt 0.250)
(d) **B1:** Writing or using N(72,72) **M1:** For exact fraction **or** awrt 83.3 (may be implied by 84) (Note: Use of N(4320,4320) can score B1 and 1st M1) **M1:** Using 84 +/– 0.5 **M1:** Standardising using 82.5, 83, $83.\dot{3}$ (awrt 83.3), 83.5, 83.8, 84 or 84.5, ‘their mean’ **and** ‘their sd’

Question	Scheme	Marks
2(a)	$P(X > 4) = 1 - F(4)$ $= 1 - \frac{3}{5}$	M1
	$= \frac{2}{5}$ oe	A1
		(2)
(b)	1	B1
		(1)
(c)	$f(x) = \frac{dF(x)}{dx} = \frac{1}{5}$	M1
	$f(x) = \begin{cases} \frac{1}{5} & 1 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$	A1
		(2)
(d)	$E(X) = 3.5$	B1
		(1)
(e)	Variance $= \frac{(6-1)^2}{12}$ or $\int_1^6 \frac{1}{5}x^2\, dx - (3.5)^2$	M1
	$= \frac{25}{12}$ awrt 2.08	A1
		(2)
(f)	$E(X^2) = Var(X) + [E(X)]^2$ $= \frac{25}{12} + 3.5^2$ or $\int_1^6 \frac{1}{5}x^2\, dx$ or $\int_1^6 \frac{1}{5}(3x^2+1)\, dx$ $= \frac{43}{3}$	M1
	$E(3X^2 + 1) = 3\,E(X^2) + 1$ $\quad = \left[\frac{3x^3}{15} + \frac{x}{5}\right]_1^6$	dM1
	$= 44$ $\quad = 44$	A1cao
		(3)
		(11 marks)

Notes:

(a)

M1: Writing or using $1 - F(4)$ o.e.

(c)

M1: For differentiating to get $\frac{1}{5}$

Question 2 notes *continued*
A1: Both lines correct with ranges
(e) **M1:** $\frac{(6-1)^2}{12}$ or $\int_1^6 \frac{1}{5}x^2\ \mathrm{d}x$ – 'their 3.5'^{2}
(f) **M1:** "Their Var(X)" + ["their E(X)"]^2 (which must follow from the 1st method in (e)) **or** $\int_1^6 \frac{1}{5}x^2\ \mathrm{d}x$ **and** integrating $x^n \to \frac{x^{n+1}}{n+1}$ (may be seen in (e)) **or** writing $\int_1^6 \frac{1}{5}(3x^2+1)\ \mathrm{d}x$ (May be implied by $\frac{43}{3}$ seen) **dM1:** **Using** 3 × 'their E(X^2)' + 1 **or** $\int_1^6 \frac{1}{5}(3x^2+1)\ \mathrm{d}x$ and integrating $x^n \to \frac{x^{n+1}}{n+1}$

Question	Scheme	Marks
3(a)	(A random variable) that is a function of a (random) sample involving no unknown quantities/parameters **or** A quantity calculated solely from a random sample	B1
		(1)
(b)	If all possible samples are chosen from a population;	B1
	then the values of a statistic and the associated probabilities is a sampling distribution	
	or a probability distribution of a statistic	
		(1)
(c)	$\text{Mean} = 100 \times \frac{4}{7} + 200 \times \frac{3}{7}$ $= \frac{1000}{7}$ awrt 143	B1
	$\text{Variance} = 100^2 \times \frac{4}{7} + 200^2 \times \frac{3}{7} - \left(\frac{1000}{7}\right)^2$	M1
	$= \frac{120000}{49}$ awrt 2450 (to 3sf)	A1
		(3)
(d)	(100,100,100)	B2
	(100,100,200) (100,200,100) (200,100,100) or 3 x (100,100,200)	
	(100,200,200) (200,100,200) (200,200,100) or 3 x (100,200,200)	
	(200,200,200)	
		(2)
(e)	(100,100,100) $\left(\frac{4}{7}\right)^3 = \frac{64}{343}$ awrt 0.187	B1 both
	(200,200,200) $\left(\frac{3}{7}\right)^3 = \frac{27}{343}$ awrt 0.0787	
	(100,100,200) $3 \times \left(\frac{4}{7}\right)^2 \times \left(\frac{3}{7}\right) = \frac{144}{343}$ awrt 0.420 (allow 0.42)	M1
	(100,200,200) $3 \times \left(\frac{4}{7}\right) \times \left(\frac{3}{7}\right)^2 = \frac{108}{343}$ awrt 0.315	A1

Question	Scheme					Marks
3(e) *continued*	m	100	$400/3$ awrt 133	$500/3$ awrt 167	200	
	$\mathrm{P}(M = m)$	$\frac{64}{343}$ or awrt 0.187	$\frac{144}{343}$ or awrt 0.420 (allow 0.42)	$\frac{108}{343}$ or awrt 0.315	$\frac{27}{343}$ or awrt 0.0787	A1
						(4)
						(11 marks)

Notes:

(a)

B1: For a definition which includes each of the following 3 aspects

A **function**[1] of a (random) **sample**[2] involving **no unknown quantities/parameters**[3]

1. function/quantity/calculation/value/random variable
2. sample/observations/data
3. no unknown parameters/no unknown values/solely (from a sample)

(b)

B1: Requires all underlined words: <u>All</u> <u>values</u> of a <u>statistic</u> with their associated <u>probabilities</u>

or

<u>probability distribution</u> of a <u>statistic</u>

(c)

M1: $100^2 \times \frac{4}{7} + 200^2 \times \frac{3}{7} - (\text{their mean})^2$

(d)

B1: Any 2 of (100,100,100), (100,100,200) any order, (100,200,200) any order, (200,200,200)

B1: All correct, allow 3 × (100,100,200) and 3 × (100,200,200) and (100,100,100) and (200,200,200)

(Note: Allow other notation for 100 and 200 e.g. Small and Large)

(e)

B1: Both probabilities for (100,100,100) and (200,200,200) correct

M1: $3 \times p^2 \times (1 - p)$

A1: Either correct

A1: All means correct **and** all probabilities correct (table not required but means must be associated with correct probabilities)

Question	Scheme	Marks
4(a)	$X \sim \text{Po}(6)$	M1
	$\text{P}(5 \le X < 7) = \text{P}(X \le 6) - \text{P}(X \le 4)$ or $\frac{e^{-6}6^5}{5!} + \frac{e^{-6}6^6}{6!}$ $= 0.6063 - 0.2851$	M1
	$= 0.3212$ awrt 0.321	A1
		(3)
(b)	H_0: $\lambda = 9$ $\quad$ H_1: $\lambda < 9$	B1
	$X \sim \text{Po}(9)$ therefore $\text{P}(X \le 4) = 0.05496...$ or CR $X \le 3$	B1
	Insufficient evidence to reject H_0 **or** Not Significant **or** 4 does not lie in the critical region.	dM1
	There is no evidence that the mean number of <u>accidents</u> at the crossroads has <u>reduced/decreased</u>.	A1cso
		(4)
		(7 marks)

Notes:

(a)

M1: Writing or using Po(6)

M1: Either $\text{P}(X \le 6) - \text{P}(X \le 4)$ or $\frac{e^{-\lambda}\lambda^5}{5!} + \frac{e^{-\lambda}\lambda^6}{6!}$

(b)

B1: Both hypotheses correct (λ or μ) allow 0.5 instead of 9

B1: Either awrt 0.055 **or** critical region $X \le 3$

dM1: For a correct comment (dependent on previous B1)

Contradictory non-contextual statements such as "not significant" so "reject H_0" score M0.

(May be implied by a correct contextual statement)

A1: Cso requires correct contextual conclusion with underlined words and all previous marks in (b) to be scored.

Question	Scheme	Marks
5(a)	$\int_{-1}^{2} k(x^2+a)\mathrm{d}x + \int_{2}^{3} 3k \ \mathrm{d}x = 1$	M1
	$\left[k\left(\frac{x^3}{3}+ax\right)\right]_{-1}^{2} + \left[3kx\right]_2^3 = 1$	dM1
	$k\left(\frac{8}{3}+2a+\frac{1}{3}+a\right)+9k-6k=1$	A1
	$6k+3ak=1$ $\int_{-1}^{2} k(x^3+ax)\mathrm{d}x + \int_{2}^{3} 3kx \ \mathrm{d}x \left[=\frac{17}{12}\right]$	M1
	$\left[k\left(\frac{x^4}{4}+\frac{ax^2}{2}\right)\right]_{-1}^{2} + \left[\frac{3kx^2}{2}\right]_2^3 = \frac{17}{12}$	dM1
	$k\left(4+2a-\frac{1}{4}-\frac{a}{2}\right)+\frac{27k}{2}-6k=\frac{17}{12}$	A1
	$\frac{45k}{4}+\frac{3ak}{2}=\frac{17}{12}$ $135k+18ak=17$ $99k = 11$	ddM1
	$a=1, k=\frac{1}{9}$	A1
		(8)
(b)	2	B1
		(1)
		(9 marks)

Notes:

(a)

M1: Writing or using $\int_{-1}^{2} k(x^2+a)\mathrm{d}x + \int_{2}^{3} 3k \ \mathrm{d}x \ = 1$ ignore limits.

dM1: Attempting to integrate at least one $x^n \to \frac{x^{n+1}}{n+1}$ **and** sight of correct limits (dependent on previous M1).

A1: Correct equation – need not be simplified.

M1: $\int_{-1}^{2} k(x^3+ax)\mathrm{d}x + \int_{2}^{3} 3kx \ \mathrm{d}x$ ignore limits.

dM1: Setting $=\frac{17}{12}$ **and** attempting to integrate at least one $x^n \to \frac{x^{n+1}}{n+1}$ **and** sight of correct limits (dependent on previous M1).

Question 5 notes *continued*
A1: A correct equation – need not be simplified. **ddM1:** Attempting to solve two simultaneous equations in *a* and *k* by eliminating 1 variable (dependent on 1st and 3rd M1s). **A1:** Both *a* and *k* correct.

Question	Scheme	Marks
6(a)	$P(X=5) = {}^{20}C_5(0.3)^5(0.7)^{15}$ or $0.4164 - 0.2375$	M1
	$= 0.17886\ldots$ awrt 0.179	A1
		(2)
(b)	Mean = 6	B1
	sd = $\sqrt{20\times0.7\times0.3}$	M1
	$= 2.049\ldots$ awrt 2.05	A1
		(3)
(c)	$H_0: p = 0.3$ $H_1: p > 0.3$	B1
	$X\sim B(20,0.3)$	M1
	$P(X \geq 8) = 0.2277$ or $P(X \geq 10) = 0.0480$, so CR $X \geq 10$	A1
	Insufficient evidence to reject H_0 **or** Not Significant **or** 8 does not lie in the critical region.	dM1
	There is no evidence to support the <u>Director (of Studies')</u> <u>belief</u>/There is no evidence that the <u>proportion</u> of <u>parents</u> that <u>do not support</u> the <u>new curriculum</u> is greater than 30%	A1 cso
		(5)
(d)	$X\sim B(2n, 0.25)$	
	$X\sim B(8, 0.25)$ $P(X \geq 4) = 0.1138$	M1
	$X\sim B(10, 0.25)$ $P(X \geq 5) = 0.0781$	
	$2n = 10$ $n = 5$	A1 A1
		(3)
		(13 marks)

Notes:

(a)

M1: ${}^{20}C_5(p)^5(1-p)^{15}$ or using $P(X \leq 5) - P(X \leq 4)$

(b)

M1: Use of $20\times0.7\times0.3$ (with or without the square root).

(c)

B1: Both hypotheses correct (p or π).

M1: Using $X\sim B(20,0.3)$ (may be implied by 0.7723, 0.2277, 0.8867 or 0.1133)

A1: Awrt 0.228 **or** CR $X \geq 10$

dM1: A correct comment (dependent on previous M1)

A1: Cso requires correct contextual conclusion with underlined words and all previous marks in (c) to be scored.

Question 6 notes *continued*
(d) **M1:** For 0.1138 or 0.0781 or 0.8862 or 0.9219 seen. **A1:** B(10, 0.25) selected (may be implied by n = 10 or $2n$ = 10 or n = 5). An answer of 5 with no incorrect working seen scores 3 out of 3. Special Case: Use of a normal approximation. **M1:** For $\frac{(n-0.5)-\frac{n}{2}}{\sqrt{\frac{3}{8}n}} = z$ with $1.28 \le z \le 1.29$, 1st A1 for n=4.2/4.3, 2nd A1 for n=5

Question	Scheme	Marks
7	$Y \sim \mathrm{N}\left(\frac{n}{5}, \frac{4n}{25}\right)$	B1
	$\mathrm{P}(Y \geq 30) = \mathrm{P}\left(Z > \dfrac{29.5 - n/5}{\frac{2}{5}\sqrt{n}}\right)$	M1 M1 A1
	$\dfrac{29.5 - n/5}{\frac{2}{5}\sqrt{n}} = 2$	B1
	$n + 4\sqrt{n} - 147.5 = 0$ or $0.04n^2 - 12.44n + 870.25 = 0$	dM1
	$\sqrt{n} = 10.3...$ $n = 106.26\ldots$ or $n = 204.73\ldots$	A1
	$n = 106$	A1 cao
		(8 marks)

Notes:

B1: Writing or using $\mathrm{N}\left(\frac{n}{5}, \frac{4n}{25}\right)$

M1: Writing or using 30 +/– 0.5

M1: Standardising using 29, 29.5, 30 or 30.5 and their mean and their sd

A1: Fully correct standardisation (allow +/-)

B1: For z = +/– 2 or awrt 2.00 must be compatible with their standardisation

dM1: (Dependent on 2nd M1) getting quadratic equation **and** solving leading to a value of $\sqrt{n}$ or n

A1: Awrt 10.3 **or** awrt (106 **or** 107 **or** 204 **or** 205)

A1: For 106 only (must reject other solutions if stated)

(Note: $\dfrac{29.5 - n/5}{\frac{2}{5}\sqrt{n}} = -2$ leading to an answer of 106 may score B1M1M1A1B0M1A1A1)

Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Statistics S3

Sample Assessment Materials for first teaching September 2018
Time: 1 hour 30 minutes

Paper Reference
WST13/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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S 5 9 7 7 0 A 0 1 2 8

Pearson

Answer ALL questions. Write your answers in the spaces provided.

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1. The names of the 720 members of a swimming club are listed alphabetically in the club's membership book. The chairman of the swimming club wishes to select a systematic sample of 40 names. The names are numbered from 001 to 720 and a number between 001 and w is selected at random. The corresponding name and every xth name thereafter are included in the sample.

(a) Find the value of w.

(1)

(b) Find the value of x.

(1)

(c) Write down the probability that the sample includes both the first name and the second name in the club's membership book.

(1)

(d) State one advantage and one disadvantage of systematic sampling in this case.

(2)

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Question 1 continued

Q1

(Total for Question 1 is 5 marks)

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2. Nine dancers, Adilzhan (A), Bianca (B), Chantelle (C), Lee (L), Nikki (N), Ranjit (R), Sergei (S), Thuy (T) and Yana (Y), perform in a dancing competition.

Two judges rank each dancer according to how well they perform. The table below shows the rankings of each judge starting from the dancer with the strongest performance.

Rank	1	2	3	4	5	6	7	8	9
Judge 1	S	N	B	C	T	A	Y	R	L
Judge 2	S	T	N	B	C	Y	L	A	R

(a) Calculate Spearman's rank correlation coefficient for these data.

(5)

(b) Stating your hypotheses clearly, test at the 1% level of significance, whether or not the two judges are generally in agreement.

(4)

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

Q2

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3. The number of accidents on a particular stretch of motorway was recorded each day for 200 consecutive days. The results are summarised in the following table.

Number of accidents	0	1	2	3	4	5
Frequency	47	57	46	35	9	6

(a) Show that the mean number of accidents per day for these data is 1.6

(1)

A motorway supervisor believes that the number of accidents per day on this stretch of motorway can be modelled by a Poisson distribution.

She uses the mean found in part (a) to calculate the expected frequencies for this model. Her results are given in the following table.

Number of accidents	0	1	2	3	4	5 or more
Frequency	40.38	64.61	r	27.57	11.03	s

(b) Find the value of r and the value of s, giving your answers to 2 decimal places.

(3)

(c) Stating your hypotheses clearly, use a 10% level of significance to test the motorway supervisor's belief. Show your working clearly.

(7)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total for Question 3 is 11 marks)

4. A farm produces potatoes. The potatoes are packed into sacks.
The weight of a sack of potatoes is modelled by a normal distribution with mean 25.6 kg and standard deviation 0.24 kg

(a) Find the probability that two randomly chosen sacks of potatoes differ in weight by more than 0.5 kg

(6)

Sacks of potatoes are randomly selected and packed onto pallets.

The weight of an empty pallet is modelled by a normal distribution with mean 20.0 kg and standard deviation 0.32 kg

Each full pallet of potatoes holds 30 sacks of potatoes.

(b) Find the probability that the total weight of a randomly chosen full pallet of potatoes is greater than 785 kg

(5)

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total for Question 4 is 11 marks)

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5. A Head of Department at a large university believes that gender is independent of the grade obtained by students on a Business Foundation course. A random sample was taken of 200 male students and 160 female students who had studied the course.

The results are summarised below.

		Male	Female
Grade	**Distinction**	18.5%	27.5%
	Merit	63.5%	60.0%
	Unsatisfactory	18.0%	12.5%

Stating your hypotheses clearly, test the Head of Department's belief using a 5% level of significance. Show your working clearly.

(12)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total for Question 5 is 12 marks)

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6. As part of an investigation, a random sample was taken of 50 footballers who had completed an obstacle course in the early morning. The time taken by each of these footballers to complete the obstacle course, x minutes, was recorded and the results are summarised by

$$\sum x = 1570 \quad \text{and} \quad \sum x^2 = 49\,467.58$$

(a) Find unbiased estimates for the mean and variance of the time taken by footballers to complete the obstacle course in the early morning.
(4)

An independent random sample was taken of 50 footballers who had completed the same obstacle course in the late afternoon. The time taken by each of these footballers to complete the obstacle course, y minutes, was recorded and the results are summarised as

$$\bar{y} = 30.9 \quad \text{and} \quad s_y^2 = 3.03$$

(b) Test, at the 5% level of significance, whether or not the mean time taken by footballers to complete the obstacle course in the early morning, is greater than the mean time taken by footballers to complete the obstacle course in the late afternoon.
State your hypotheses clearly.
(7)

(c) Explain the relevance of the Central Limit Theorem to the test in part (b).
(1)

(d) State an assumption you have made in carrying out the test in part (b).
(1)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total for Question 6 is 13 marks)

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7. A fair six-sided die is labelled with the numbers 1, 2, 3, 4, 5 and 6
The die is rolled 40 times and the score, S, for each roll is recorded.

(a) Find the mean and the variance of S.

(2)

(b) Find an approximation for the probability that the mean of the 40 scores is less than 3

(3)

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Question 7 continued

Q7

(Total for Question 7 is 5 marks)

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8. A factory produces steel sheets whose weights Xkg, are such that $X \sim N(\mu, \sigma^2)$

A random sample of these sheets is taken and a 95% confidence interval for μ is found to be (29.74, 31.86)

(a) Find, to 2 decimal places, the standard error of the mean.

(3)

(b) Hence, or otherwise, find a 90% confidence interval for μ based on the same sample of sheets.

(3)

Using four different random samples, four 90% confidence intervals for μ are to be found.

(c) Calculate the probability that at least 3 of these intervals will contain μ.

(3)

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Question 8 continued

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Question 8 continued

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Q8

(Total for Question 8 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

Statistics S3 Mark scheme

Question	Scheme	Marks
1(a)	$\{w\} = 018$ or 18	B1
		(1)
(b)	$\{x\} = 18$	B1
		(1)
(c)	{prob} = 0	B1
		(1)
(d)	**Advantage:** Any one of: • Simple or easy to use also allow "quick" or "efficient" (o.e.) • It is suitable for large samples (or populations) • Gives a good spread of the data	B1
	Disadvantage: Any one of: • The alphabetical list is (probably) not random • Biased since the list is not (truly) random • Some combinations of names are not possible	B1
		(2)
		(5 marks)
Notes:		
(d)	If no labels are given treat the 1st reason as an advantage and the 2nd as a disadvantage	
B1:	For advantage	
B1:	For disadvantage − "it requires a sampling frame" is 2nd B0 since the alphabetical list is given. Note: Do not score both B1 marks for opposing advantages and disadvantages.	

Question	Scheme		Marks
2(a)		*A* *B* *C* *L* *N* *R* *S* *T* *Y* Judge 1: 6 3 4 9 2 8 1 5 7 Judge 2: 8 4 5 7 3 9 1 2 6 or *S* *N* *B* *C* *T* *A* *Y* *R* *L* Judge 1: 1 2 3 4 5 6 7 8 9 Judge 2: 1 3 4 5 2 8 6 9 7	M1
	$\sum d^2 = 4+1+1+4+1+1+0+9+1$ or $0+1+1+1+9+4+1+1+4 = 22$	$\sum d^2 = 22$	M1 A1
	$r_s = 1 - \frac{6(22)}{9(80)}$; $= 0.8166666...$	$\frac{49}{60}$ or awrt **0.817**	M1 A1
			(5)
(b)	$\mathrm{H}_0 : \rho = 0$, $\mathrm{H}_1 : \rho > 0$		B1
	Critical Value = 0.7833 <u>or</u> CR: $r_s \geqslant 0.7833$	**0.7833**	B1
	Since $r_s = 0.8166...$ it lies in the CR, <u>or</u> reject H_0 (o.e.)		M1
	The two <u>judges</u> (or "they") are in <u>agreement</u> <u>or</u> there is a <u>positive correlation</u> between the ranks of the two <u>judges</u>.		A1ft
			(4)
			(9 marks)

Notes:

(a)

M1: For an attempt to rank at least one row (at least 4 correct)

M1: For an attempt at d^2 row (may be implied by sight of $\sum d^2 = 22$ or 221 for reverse ranks)

A1: For $\sum d^2 = 22$ (or 221 if reverse ranking is used) Can be implied by correct answer.

M1: For use of the correct formula with their $\sum d^2$ (if it is clearly stated)

If the answer is not correct then a correct expression is required

False Ranking - e.g. Alphabetic ranking: Gives

Judge 1: 7 5 2 3 8 1 9 6 4

Judge 2: 7 8 5 2 3 9 4 1 6 $\sum d^2 = 162$ and $r_s = -0.35$

Question 2 notes *continued*	
	Scores: M0(for ranking), M1(for attempt at d^2 row), A0, M1 (for use of their $\sum d^2$), A0 i.e. 2 out of 5. Can follow through their r_s in (b)
(b)	
B1:	For both hypotheses stated correctly in terms of ρ (allow ρ_s) H_1 must be compatible with ranking.
B1:	For cv = 0.7833 (independent of their H_1 (no 2-tail value in tables) <u>but</u> compatible sign with their r_s).
M1:	For a correct statement (in words) relating their r_s with their critical value. E.g. "reject H_0 ","in critical region", "significant", "positive correlation". May be implied by a correct contextual comment. **|cv|>1** - If their cv is |cv| > 1 (often from using normal tables) award M0A0 - If |their | > |their cv| then "significant" (o.e.) for M1 and "judges are in - agreement" (o.e.) for A1ft - If |their | < |their cv| then "not significant" (o.e.) for M1 and "judges don't agree" (o.e.) for A1ft
A1ft:	For a correct follow through conclusion in context. "Positive correlation" alone scores M1 A0. For reverse ranking should still say " judges <u>are</u> in agreement"

Question	Scheme		Marks
3(a)	$\lambda = \dfrac{0(47)+1(57)+2(46)+3(35)+4(9)+5(6)}{200} = \dfrac{320}{200} = 1.6$ *	Full exp' or at least 2 products and 320/200 seen	B1 *
			(1)
(b)	$r = 200 \times \dfrac{e^{-1.6}(1.6)^2}{2!}$ $\{= 51.68550861...\}$	Using $r = 200 \times \dfrac{e^{-1.6}(1.6)^2}{2!}$	M1
	$s = 200 - (40.38 + 64.61 + \text{their } r + 27.57 + 11.03)$ $\{= 4.72449139...\}$ <u>or</u> their $r + s = 56.41$		M1
	$r = 51.68550861...$ and $s = 4.72449139...$	$r =$ awrt **51.69** and $s =$ awrt **4.72**	A1
			(3)
(c)	H_0 : Poisson (distribution) is a suitable/ sensible (model) H_1 : Poisson (distribution)is not a suitable/ sensible (model).		B1

Number of accidents	Observed	Expected	Combined Observed	Combined Expected	$\frac{(O-E)^2}{E}$	$\frac{O^2}{E}$
0	47	40.38	47	40.38	1.0853	54.7053
1	57	64.61	57	64.61	0.8963	50.2863
2	46	51.69	46	51.69	0.6264	40.9364
3	35	27.57	35	27.57	2.0024	44.4324
4	9	11.03	15	15.75	0.0357	14.2857
$\geqslant 5$	6	4.72				
				Totals	4.6461	204.6461

M1

Question	Scheme		Marks
	$X^2 = \sum \dfrac{(O-E)^2}{E}$ or $\sum \dfrac{O^2}{E} - 200$;$= 4.6461$		M1
		awrt **4.65**	A1
	$v = 5 - 1 - 1 = 3$	**3**	B1 ft
	$\chi_3^2(0.10) = 6.251 \Rightarrow$ CR: $X^2 \geqslant 6.251$	**6.251**	B1 ft
	[Since $X^2 = 4.6461$ does not lie in the CR, then there is insufficient evidence to reject H_0]		
	The number of ***accidents*** per day can be modelled by a Poisson distribution <u>or</u> the ***supervisor's*** belief is correct.		A1 ft
			(7)
			(11 marks)

Notes:

(b)

Note: Allow A1 for $s =$ awrt 4.74 (fou as a result of using expected values to full accuracy.)

Question 3 notes *continued*
(c) **B1:** For <u>both</u> hypotheses and mentioning Poisson at least once. Allow Poisson is a "good fit/model" but <u>not</u> "good method". Inclusion of 1.6 for mean in hypotheses is B0 but condone in conclusion. **M1:** For an attempt to pool 4 accidents and $\geqslant 5$ accidents <u>or</u> pool when $E_i < 5$ No pooling is M0 **M1:** For an attempt at the test statistic, at least 2 correct expressions/values (to awrt 2 d.p.) **A1:** For awrt 4.65 (score M1M1A1 if awrt 4.65 seen). **No pooling:** If no pooling can allow 2nd M1 if $X^2 = 5.33$ is seen **B1ft:** For $n-1-1$ i.e. subtracting 2 from their n. **B1ft:** For a correct ft for their $\chi_k^2(0.10)$, where $k = n-1-1$ from their n. (B1B1 may be implied by 6.251 (if pooling) or 7.779 for no pooling) **A1ft:** (***Dep. on the*** 2nd M1) For correct comment in context based on their test statistic and their critical value that mentions ***accidents*** or ***supervisor***. Condone mention of Po(1.6) in conclusion. Score A0 for inconsistencies e.g. "significant" followed by "supervisor's belief is justified" Note: Full accuracy gives a combined expected frequency of 15.76, $\frac{(O-E)^2}{E} = 0.0366$, $\frac{O^2}{E} = 14.2766$, $X^2 = 4.64855...$ and p-value 0.199.

Question	Scheme		Marks
4(a)	Let X = weight of a sack of potatoes, $X \sim \mathrm{N}(25.6\,,\,0.24^2)$		
	So $D = X_1 - X_2 \sim \mathrm{N}(0\,,\,2(0.24)^2)$ or $D \sim \mathrm{N}(0\,,\,0.1152)$	Attempt at D and $D \sim \mathrm{N}(0, ..)$	M1
		$(0.24)^2 + (0.24)^2$; **0.1152**	A1 A1
	$\{\mathrm{P}(\lvert D\rvert > 0.5) =\}\ 2\mathrm{P}(D > 0.5)$	$2 \times \mathrm{P}(D > 0.5)$ can be implied	dM1
	$= 2 \times \mathrm{P}\left(Z > \frac{0.5}{\sqrt{0.1152}}\right)$		dM1
	$= 2 \times \mathrm{P}(Z > 1.4731...)$ <u>or</u> $= 2(1 - 0.9292)$		
	$= 0.1416$	**awrt 0.141** or **awrt 0.142**	A1
			(6)
(b)	Let Y = weight of an empty pallet, $Y \sim \mathrm{N}(20.0\,,\,0.32^2)$		
	So $T = X_1 + X_2 + ... + X_{30} + Y$		
	$T \sim \mathrm{N}(30(25.6) + 20,\ 30(0.24)^2 + 0.32^2)$	$30(25.6) + 20$ <u>or</u> **788**	B1
		$30(0.24)^2 + 0.32^2$	M1
	$T \sim \mathrm{N}(788, 1.8304)$	N and 1.8304 or awrt **1.83**	A1
	$\{\mathrm{P}(T > 785) =\}\ \mathrm{P}\left(Z > \frac{785 - 788}{\sqrt{1.8304}}\right)$		M1
	$= \mathrm{P}(Z > -2.2174...)$		
	$= 0.9868$	**awrt 0.987**	A1
			(5)
			(Total 11)

Notes:

(a)

M1: For clear definition of D and normal distribution with mean of 0 (Can be implied by 3rd M1).

A1: For correct use of Var($X_1 - X_2$) formula.

A1: For 0.1152

dM1: For realising need $2 \times \mathrm{P}(D > 0.5)$ (Dependent on 1st M1 i.e. must be using suitable D).

dM1: Dep on 1st M1 for standardising with 0.5, 0 and their s.d.($\neq 0.24$)Must lead to $\mathrm{P}(Z > +\text{ve})$ (o.e.). P(Z > 1.47) implies 1st M1 1st A1 2nd A1 and 3rd M1. Correct answer only will score 6 out of 6.

Question 4 notes *continued*	
(b)	
B1:	For a mean of $30(25.6) + 20$. Can be implied by 788.
M1:	For $30(0.24)^2 + 0.32^2$. Can be implied by 1.8304 or awrt 1.83
	Allow M1 for swapping error i.e. $30 \times 0.32^2 + 0.24^2$ if the expression is seen
A1:	For normal and correct variance of 1.8304 or awrt 1.83. Normality may be implied by standardisation
M1:	For standardising with 785 with their mean and st. dev..($\neq 0.24$) Must lead to $\mathrm{P}(Z > -\text{ve})$ o.e.
A1:	Awrt 0.987. Correct answer only will score 5 out of 5
	Note: Calculator answers are (a) 0.14071... , (b) 0.98670...

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td>5</td><td colspan="2"><u>Mark Scheme for candidates who use percentages instead of observed values.</u></td><td></td></tr>
<tr><td></td><td>H_0 : Grades and gender are independent (or not associated)
H_1 : Grades and gender are dependent (or associated)</td><td>“grades” and “gender” mentioned at least once.</td><td>B1</td></tr>
<tr><td></td><td>
<table>
<tr><th>Observed</th><th>Male</th><th>Female</th></tr>
<tr><td>Distinction</td><td>18.5</td><td>27.5</td></tr>
<tr><td>Merit</td><td>63.5</td><td>60.0</td></tr>
<tr><td>Unsatisfactory</td><td>18.0</td><td>12.5</td></tr>
</table>
</td><td>These marks cannot be obtained.</td><td>M0 A0</td></tr>
<tr><td></td><td rowspan="2">
<table>
<tr><th>Expected</th><th>Male</th><th>Female</th><th>Totals</th></tr>
<tr><td>Distinction</td><td>23</td><td>23</td><td>46</td></tr>
<tr><td>Merit</td><td>61.75</td><td>61.75</td><td>123.5</td></tr>
<tr><td>Unsatisfactory</td><td>15.25</td><td>15.25</td><td>30.5</td></tr>
<tr><td>Totals</td><td>100</td><td>100</td><td>200</td></tr>
</table>
</td><td>Some attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$
Can be implied by one of these E_i ’s</td><td>M1</td></tr>
<tr><td></td><td>Expected frequencies are not correct.</td><td>A0</td></tr>
<tr><td></td><td rowspan="2">
<table>
<tr><th>Observed</th><th>Expected</th><th>$\frac{(O-E)^2}{E}$</th><th>$\frac{O^2}{E}$</th></tr>
<tr><td>18.5</td><td>23</td><td>0.8804</td><td>14.8804</td></tr>
<tr><td>27.5</td><td>23</td><td>0.8804</td><td>32.8804</td></tr>
<tr><td>63.5</td><td>61.75</td><td>0.0496</td><td>65.2996</td></tr>
<tr><td>60.0</td><td>61.75</td><td>0.0496</td><td>58.2996</td></tr>
<tr><td>18.0</td><td>15.25</td><td>0.4959</td><td>21.2459</td></tr>
<tr><td>12.5</td><td>15.25</td><td>0.4959</td><td>10.2459</td></tr>
<tr><td colspan="2">Totals</td><td>2.8518</td><td>202.8518</td></tr>
</table>
</td><td>At least 2 “correct” terms for $\frac{(O-E)^2}{E}$ or $\frac{O^2}{E}$ or correct expressions with their E_i .
Accept 2 sf accuracy for the M1 mark.</td><td>M1</td></tr>
<tr><td></td><td>This mark cannot be obtained.</td><td>A0</td></tr>
<tr><td></td><td>$X^2 = \sum \frac{(O-E)^2}{E}$ or $\sum \frac{O^2}{E} - 360$;= 2.8518</td><td>This mark cannot be obtained.</td><td>A0</td></tr>
<tr><td></td><td>$\nu = (3-1)(2-1) = 2$</td><td>$(\nu =)$ 2 (Can be implied by 5.991)</td><td>B1</td></tr>
<tr><td></td><td>$\chi_2^2(0.05) = 5.991 \Rightarrow$ CR: $X^2 \geqslant 5.991$</td><td>For 5.991 only</td><td>B1</td></tr>
</table>

Question	Scheme		Marks
5 ***continued***	Since $X^2 = 2.86$ does not lie in the CR, then there is insufficient evidence to reject H_0		M1
		Not available since comes from incorrect	A0
			(12)
			(12 marks)

Notes:

If a candidate uses percentages rather than observed values then they can obtain a maximum of **6 marks**. They can get B1 M0A0 M1A0 M1A0A0 B1B1M1A0.

Question	Scheme		Marks
6(a)	$\left\{\hat{\mu} = \frac{\sum x}{n} = \frac{1570}{50} = \right\}$ $\bar{x} = 31.4$	$\bar{x} =$ **31.4**	B1 cao
	$\left\{\hat{\sigma}^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \right\}$ $s_x^2 = \frac{49467.58 - 50(31.4)^2}{50-1}$		M1 A1ft
	$= 3.460816...$	awrt **3.46**	A1
			(4)
(b)	[Let Y = time taken to complete obstacle course in the afternoon.]		
	$H_0 : \mu_x = \mu_y$, $H_1 : \mu_x > \mu_y$		B1
	$(z =) \frac{"31.4" - 30.9}{\sqrt{\frac{"3.46"}{50} + \frac{3.03}{50}}}$		M1 A1ft
	$= 1.38781...$	awrt **1.39**	A1
	CR: $Z \geqslant 1.6449$ or probability = awrt 0.082 or awrt 0.083	**1.6449** or better	B1
	Since $z = 1.38781...$ does not lie in the CR, then there is insufficient evidence to reject H_0		M1
	Conclude that the <u>mean time</u> to complete the obstacle course is the same for the early <u>morning</u> and late <u>afternoon</u>.		A1
			(7)
(c)	$\bar{X}$ <u>and</u> $\bar{Y}$ are both approx. <u>normal</u>ly distributed <u>or</u> $\bar{X} - \bar{Y}$ normal (Condone $\bar{x}$ and $\bar{y}$)		B1
			(1)
(d)	Have assumed $s^2 \simeq \sigma^2$ or variance of sample $\simeq$ variance of population		B1
			(1)
			(13 marks)

Notes:

(a)

B1: 31.4 cao. Allow 31 minutes, 24 seconds.

M1: A correct expression for either s or s^2 (ignore label)

A1ft: A correct expression for s^2 with their ft $\bar{x}$.

A1: Awrt 3.46 (Correct answer scores 3 out of 3)

(b)

B1: Both hypotheses stated correctly, with some indication of which μ is which. E.g: μ_M , μ_A

Question 6 notes *continued*
M1: For an attempt at $\dfrac{a-b}{\sqrt{\dfrac{c}{50}+\dfrac{d}{50}}}$ with at least 3 of *a, b, c* or *d* correct. Allow $\pm$ **A1ft:** For $\pm\dfrac{\text{their } 31.4-30.9}{\sqrt{\dfrac{\text{their } 3.46}{50}+\dfrac{3.03}{50}}}$ Allow $D=\bar{x}-\bar{y}$ $\quad 1.64 \sim 1.65 = \dfrac{D-0}{\sqrt{\dfrac{"3.46"}{50}+\dfrac{3.03}{50}}}$ [SE = 0.360277..]   **A1:** For awrt 1.39 (possibly $\pm$)(Allow for CV D = awrt 0.593) (NB d = 0.5)   Correct answer scores M1A1ftA1 <u>but</u> $0-(31.4-30.9)\rightarrow -1.39$ loses this 2nd A mark   **B1:** Critical value of 1.6449 or better (seen). Allow for probability = awrt 0.082 or awrt 0.083.   Note: p-values are 0.0823 (tables) and 0.0826 (calculator).   **M1:** For a correct statement linking their test statistic and their critical value.   Note: Contradictory statements score M0. E.g. "significant, do not reject H_0".   **A1:** For a correct statement in context that accepts H_0 (no ft) Condone "no difference in mean times". Must mention "<u>mean time</u>", "<u>morning</u>" and "<u>afternoon</u>" or "<u>both</u> <u>times</u> <u>of</u> <u>day</u>"
(c)   **B1:** E.g. $\bar{X} \sim N(...)$ need both. Allow in words e.g "sample means are normally distributed".
(d)   **B1:** Condone only mentioning "*x*" or "*y*" <u>but</u> watch out for $s_x = s_y$ or $\sigma_x = \sigma_y$ which scores B0.

Question	Scheme		Marks
7(a)	Let X = score on a die		
	$\text{E}(S) = 3.5$, $\text{Var}(S) = \dfrac{35}{12}$	$\text{E}(S) =$ **3.5**	B1
		$\text{Var}(S) = \dfrac{35}{12}$ or awrt **2.92**	B1
			(2)
(b)	So, $\bar{S} \sim \text{N}\left(\text{"3.5"}, \dfrac{\text{"}\left(\frac{35}{12}\right)\text{"}}{40}\right)$ or $\bar{S} \sim \text{N}\left(\text{"3.5"}, \dfrac{7}{96}\right)$		B1ft
	$\text{P}\left(\bar{S} < 3\right) = \text{P}\left(Z < \dfrac{3 - \text{"3.5"}}{\sqrt{\frac{7}{96}}}\right)$ $\{= \text{P}\left(Z < -1.85164...\right)\}$		M1
	$\{= 1 - 0.9678\} = 0.0322$	**0.032 to 0.0322**	A1
			(3)
			(5 marks)

Notes:

(a)

B1: (2^{nd}) allow awrt 2.92

(b)

B1ft: For $\bar{S} \sim \text{N}\left(\text{"3.5"}, \dfrac{\text{"}\left(\frac{35}{12}\right)\text{"}}{40}\right)$ seen or implied. Follow through their E(S) and their Var(S)

N.B $\dfrac{7}{96} = 0.07291\dot{6}$ accept awrt 0.0729

M1: For an attempt to standardise with 3, their mean (>3) and $\sqrt{\dfrac{\text{their Var}(S)}{40}}$. Must lead to $\text{P}(Z < -\text{ve})$

A1: For 0.032 ~ 0.0322

Alternative ΣS

B1ft: For $\sum S \sim \text{N}\left(140, \dfrac{350}{3}\right)$ where 140 is $40 \times$ their E(S) and variance is $40 \times$ their Var(S).

Question 7 notes *continued*
M1: For $\mathrm{P}\left(Z < \frac{120 - "140"}{\sqrt{\frac{350}{3}}}\right)$ or $\mathrm{P}\left(Z < \frac{119.5 - "140"}{\sqrt{\frac{350}{3}}}\right)$ $\{= \mathrm{P}(Z < -1.8979...)\}$ **A1:** for 0.032~0.0322 or (with continuity correction) 0.0287 (tables) or 0.0289 (calculator).

Question	Scheme		Marks
8(a)	$\left\{\bar{x} = \frac{29.74 + 31.86}{2}\right\} \Rightarrow \bar{x} = 30.8$	$\bar{x} = 30.8$ This can be implied. See note.	B1
	$"1.96"\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 30.8$ or $2("1.96")\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 29.74$		M1
	$\text{SE}_{\bar{x}} = \frac{31.86 - 30.8}{1.96} = 0.540816... = 0.54\ (2\,\text{dp})$	awrt **0.54**	A1
			(3)
(b)	A 90% CI for μ is $\bar{x} \pm 1.6449\left(\frac{\sigma}{\sqrt{n}}\right)$		B1
	$= 30.8 \pm 1.6449(0.54)$	$(\text{their } \bar{x}) \pm (\text{their } z)(\text{their SE}_{\bar{x}} \text{ from (a)})$	M1
	$= (29.91,\ 31.69)$	(awrt **29.9**, awrt **31.7**)	A1
			(3)
(c)	Let X = number of confidence intervals containing μ		
	or Y = number of confidence intervals not containing μ		
	So $X \sim \text{Bin}(4, 0.9)$ or $Y \sim \text{Bin}(4, 0.1)$		M1
	$\text{P}(X \geqslant 3)$ or $\text{P}(Y \leqslant 1) = {}^4C_3(0.9)^3(0.1) + (0.9)^4$	${}^4C_3(0.9)^3(0.1) + (0.9)^4$ oe	A1
	$= 0.2916 + 0.6561 = 0.9477$	**0.9477** or **0.948**	A1
			(3)
			(9 marls)

Notes:

(a)

B1: $\bar{x} = 30.8$ may be implied by $1.96\left(\frac{\sigma}{\sqrt{n}}\right) = [31.86 - 30.8] = 1.06$ <u>or</u>

$2(1.96)\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 29.74$

M1: A correct equation for either a width or a half-width involving a z-value $1.5 \leqslant z \leqslant 2$

Eg: $"\text{their } z"\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - "30.8"$ ft their $\bar{x}$ <u>or</u> $2("\text{their } z")\left(\frac{\sigma}{\sqrt{n}}\right) = 31.86 - 29.74$

or $"\text{their } z"(\text{SE}_{\bar{x}}) = 31.86 - "30.8"$ <u>or</u> $2("\text{their } z")(\text{SE}_{\bar{x}}) = 31.86 - 29.74$ are fine for M1.

A1: 0.54 or awrt 0.54 Must be seen as final answer to (a) NB $\frac{53}{98}$ as final answer is A0

Condone $\bar{x} \pm 1.96\sigma = ...$ for B1 and M1 but A0 even if they say "σ = standard error = 0.54". Otherwise answer only of 0.54 scores 3 out of 3

Question 8 notes *continued*
(b) **B1:** For use of 1.6449 or better in an attempt at a CI formula. Need at least 1.6449 (their SE). **M1:** For attempt at CI ft their values and provided $1 \leqslant z \leqslant 1.7$
(c) **M1:** States or applies either $X \sim \text{Bin}(4, 0.9)$ <u>or</u> $Y \sim \text{Bin}(4, 0.1)$ Condone M1 for $0.9^4 + 0.9^3 \times 0.1$ (o.e.) **A1:** ${}^4C_3(0.9)^3(0.1) + (0.9)^4$ or $(0.9)^4 + {}^4C_1(0.1)(0.9)^3$ oe **A1:** 0.9477 or 0.948

Pearson Edexcel International Advanced Level

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

Sample Assessment Materials for first teaching September 2018 **Time: 1 hour 30 minutes**	Paper Reference **WDM11/01**

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- There are 6 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Write your answers in the D1 answer book for this paper.

1. The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

Liz must visit each town at least once. She will start and finish at A and wishes to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree given in your answer book, use the shortcut method to find an upper bound below 810 km for Liz's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Liz's route.

(2)

(c) Starting by deleting F, and all of its arcs, find a lower bound for the length of Liz's route.

(3)

(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(1)

(Total for Question 1 is 8 marks)

2. Kruskal's algorithm finds a minimum spanning tree for a connected graph with n vertices.

(a) Explain the terms

(i) connected graph,

(ii) tree,

(iii) spanning tree.

(3)

(b) Write down, in terms of n, the number of arcs in the minimum spanning tree.

(1)

The table below shows the lengths, in km, of a network of roads between seven villages, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	17	–	19	30	–	–
B	17	–	21	23	–	–	–
C	–	21	–	27	29	31	22
D	19	23	27	–	–	40	–
E	30	–	29	–	–	33	25
F	–	–	31	40	33	–	39
G	–	–	22	–	25	39	–

(c) Complete the drawing of the network on Diagram 1 in the answer book by adding the necessary arcs from vertex C together with their weights.

(2)

(d) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(e) State the weight of the minimum spanning tree.

(1)

(Total for Question 2 is 10 marks)

3. 12.1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 33

(3)

The list is to be sorted into **descending** order.

(b) (i) Starting at the left-hand end of the list, perform two passes through the list using a bubble sort. Write down the state of the list that results at the end of each pass.

(ii) Write down the total number of comparisons and the total number of swaps performed during your two passes.

(4)

(c) Use a quick sort on the **original** list to obtain a fully sorted list in **descending** order. You must make your pivots clear.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how the numbers listed can be packed into bins of size 33

(3)

(e) Determine whether your answer to (d) uses the minimum number of bins. You must justify your answer.

(1)

(Total for Question 3 is 15 marks)

4.

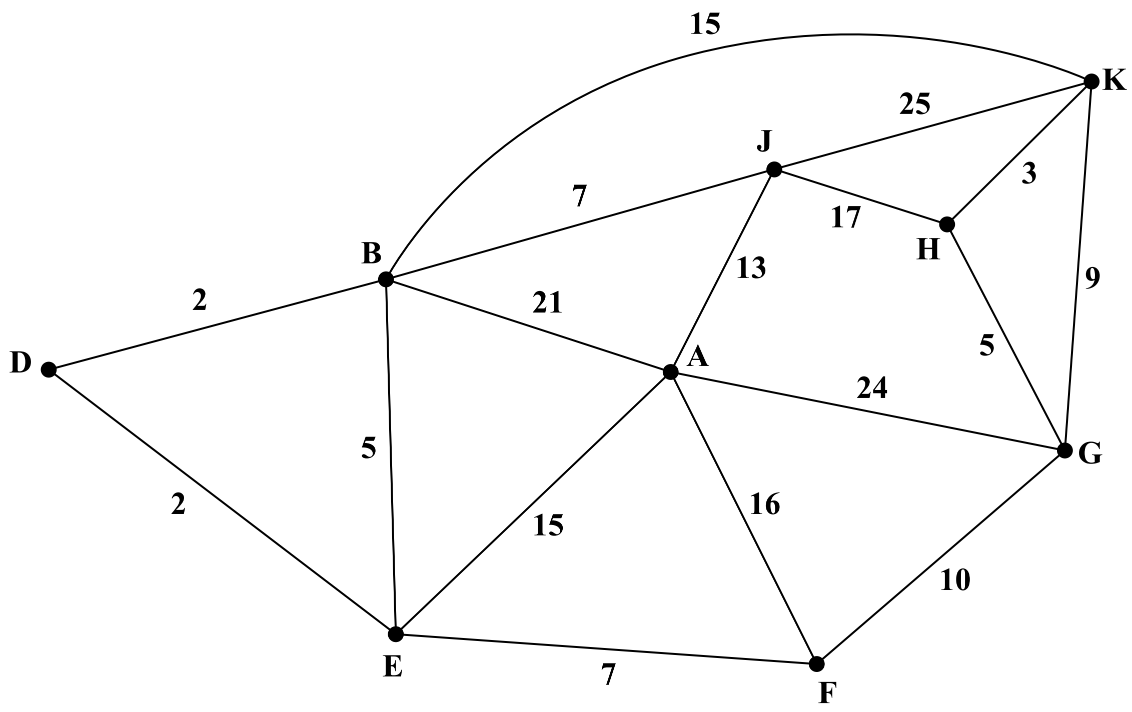

Figure 1

[*The total weight of the network is 196*]

Figure 1 models a network of roads. The number on each edge gives the time, in minutes, taken to travel along that road. Oliver wishes to travel by road from A to K as quickly as possible.

(a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to K. State the quickest route.

(6)

On a particular day Oliver must travel from B to K via A.

(b) Find a route of minimal time from B to K that includes A, and state its length.

(2)

Oliver needs to travel along each road to check that it is in good repair. He wishes to minimise the total time required to traverse the network.

(c) Use the route inspection algorithm to find the shortest time needed. You must state all combinations of edges that Oliver could repeat, making your method and working clear.

(7)

(Total for Question 4 is 15 marks)

5. A linear programming problem in x and y is described as follows.

Maximise $P = 5x + 3y$

subject to: $x \geqslant 3$

$x + y \leqslant 9$

$15x + 22y \leqslant 165$

$26x - 50y \leqslant 325$

(a) Add lines and shading to Diagram 2 in the answer book to represent these constraints. Hence determine the feasible region and label it R.

(4)

(b) Use the objective line method to find the optimal vertex, V, of the feasible region. You must draw and label your objective line and label vertex V clearly.

(2)

(c) Calculate the exact coordinates of vertex V and hence calculate the corresponding value of P at V.

(3)

The objective is now to **minimise** $5x + 3y$, where x and y are **integers**.

(d) Write down the minimum value of $5x + 3y$ and the corresponding value of x and corresponding value of y.

(2)

(Total for Question 5 is 11 marks)

6.

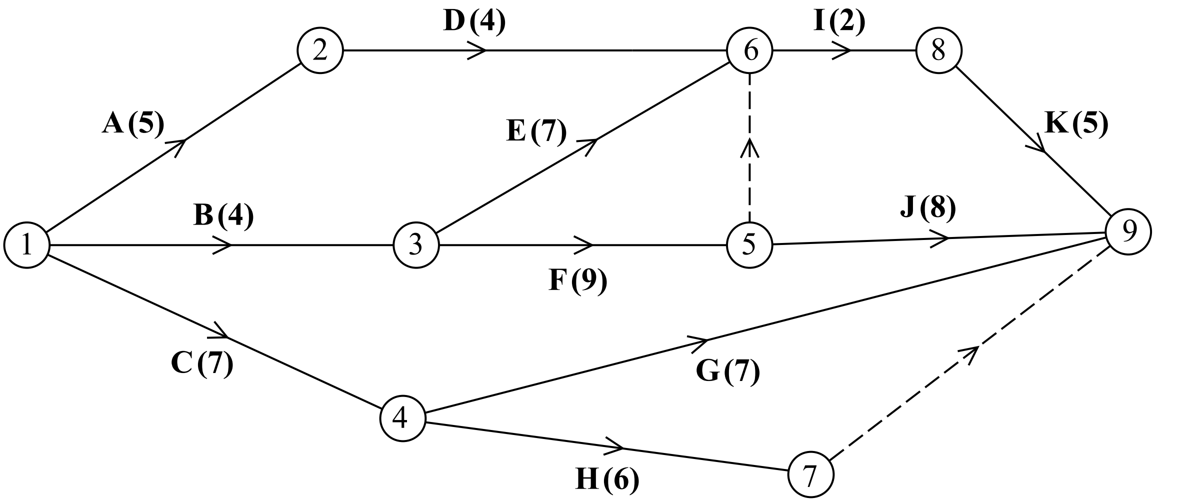

Figure 2

A project is modelled by the activity network shown in Figure 2. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.

(a) Explain the significance of the dummy activity

(i) from event 5 to event 6

(ii) from event 7 to event 9.

(2)

(b) Complete Diagram 3 in the answer book to show the early event times and the late event times.

(4)

(c) State the minimum project completion time.

(1)

(d) Calculate a lower bound for the minimum number of workers required to complete the project in the minimum time. You must show your working.

(2)

(e) On Grid 1 in your answer book, draw a cascade (Gantt) chart for this project.

(4)

(f) On Grid 2 in your answer book, construct a scheduling diagram to show that this project can be completed with three workers in just one more hour than the minimum project completion time.

(3)

(Total for Question 6 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS

Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number

Candidate Number

Mathematics

International Advanced Subsidiary/Advanced Level
Decision Mathematics D1

Sample Assessment Materials for first teaching September 2018

Time: 1 hour 30 minutes

Paper Reference
WDM11/01

Answer Book
Do not return the question paper with the answer book.

Total Marks

Turn over

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S 5 9 7 7 1 A 0 1 1 5

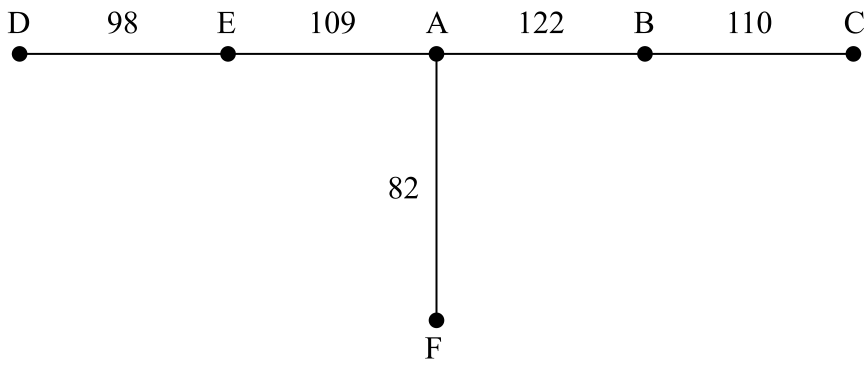
Pearson

Leave blank

1.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

(a)



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DO NOT WRITE IN THIS AREA

Question 1 continued

(b)

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

(c)

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

Q1

(Total for Question 1 is 8 marks)

Leave blank

2.

(c)

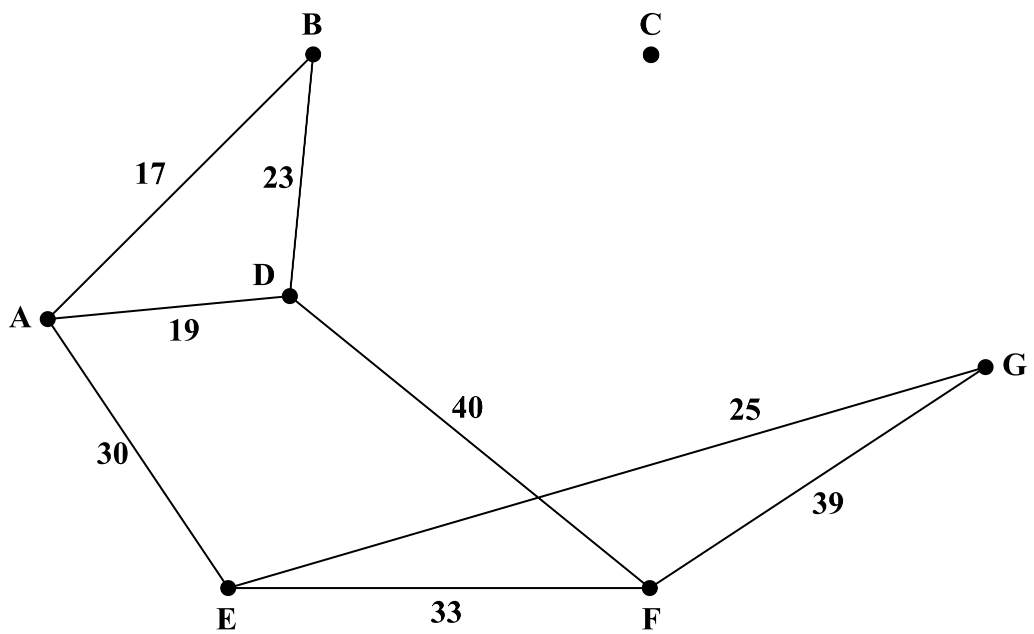

Diagram 1

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Leave blank

Question 2 continued

Q2

(Total for Question 2 is 10 marks)

Leave blank

3.

12.1	9.3	15.7	10.9	17.4	6.4	20.1	7.9	8.1	14.0

DO NOT WRITE IN THIS AREA

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Question 3 continued

12.1	9.3	15.7	10.9	17.4	6.4	20.1	7.9	8.1	14.0

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Leave blank

Question 3 continued

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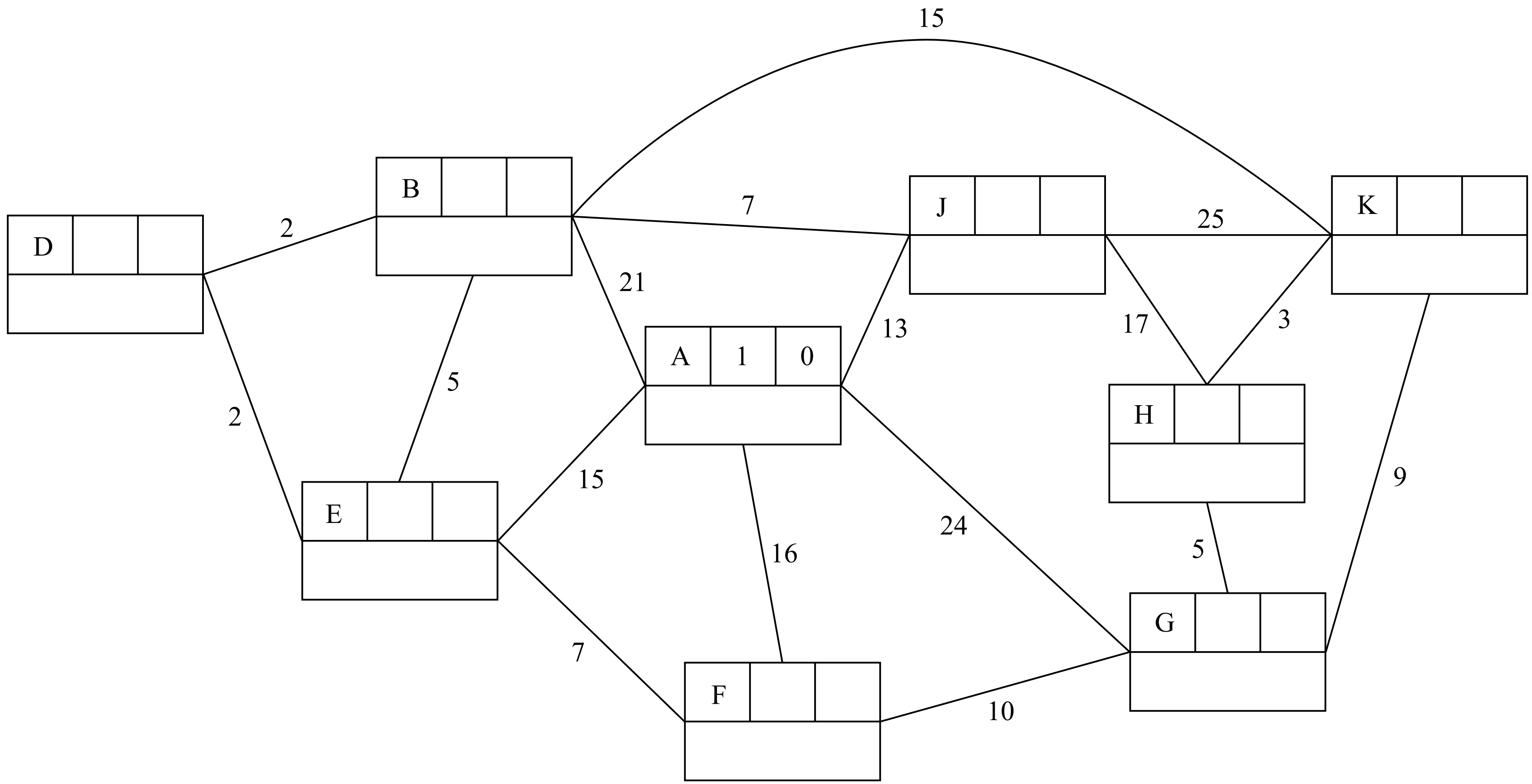
Question 3 continued

Leave blank

Q3

(Total for Question 3 is 15 marks)

4.



Quickest route: ______________________

Shortest time: ______________________

Key:

Vertex	Order of labelling	Final value
Working values		

Leave blank

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

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Question 4 continued

Q4

(Total for Question 4 is 15 marks)

Leave blank

5.

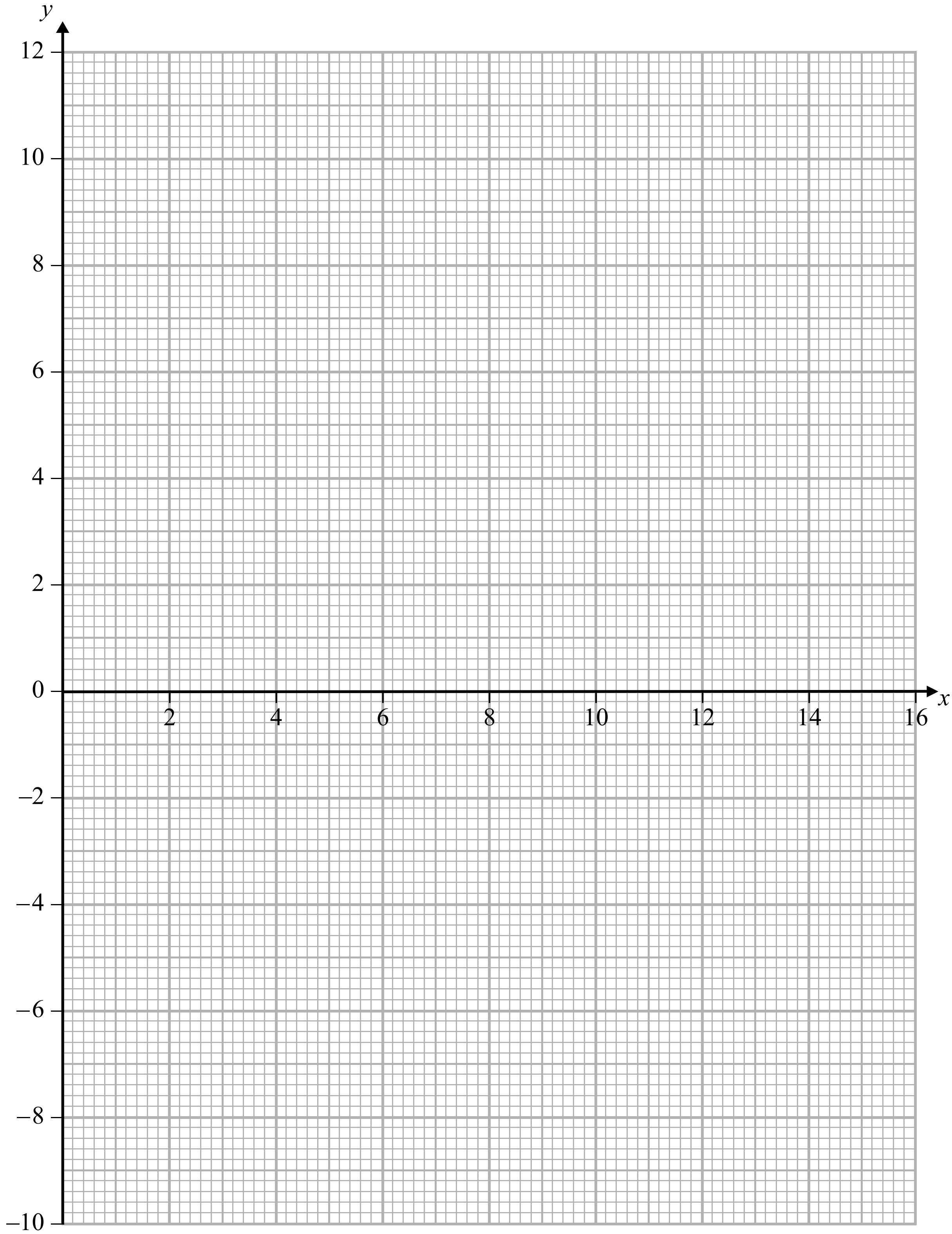


Diagram 2

Leave blank

Question 5 continued

Q5

(Total for Question 5 is 11 marks)

Leave blank

6. (a)

(b)

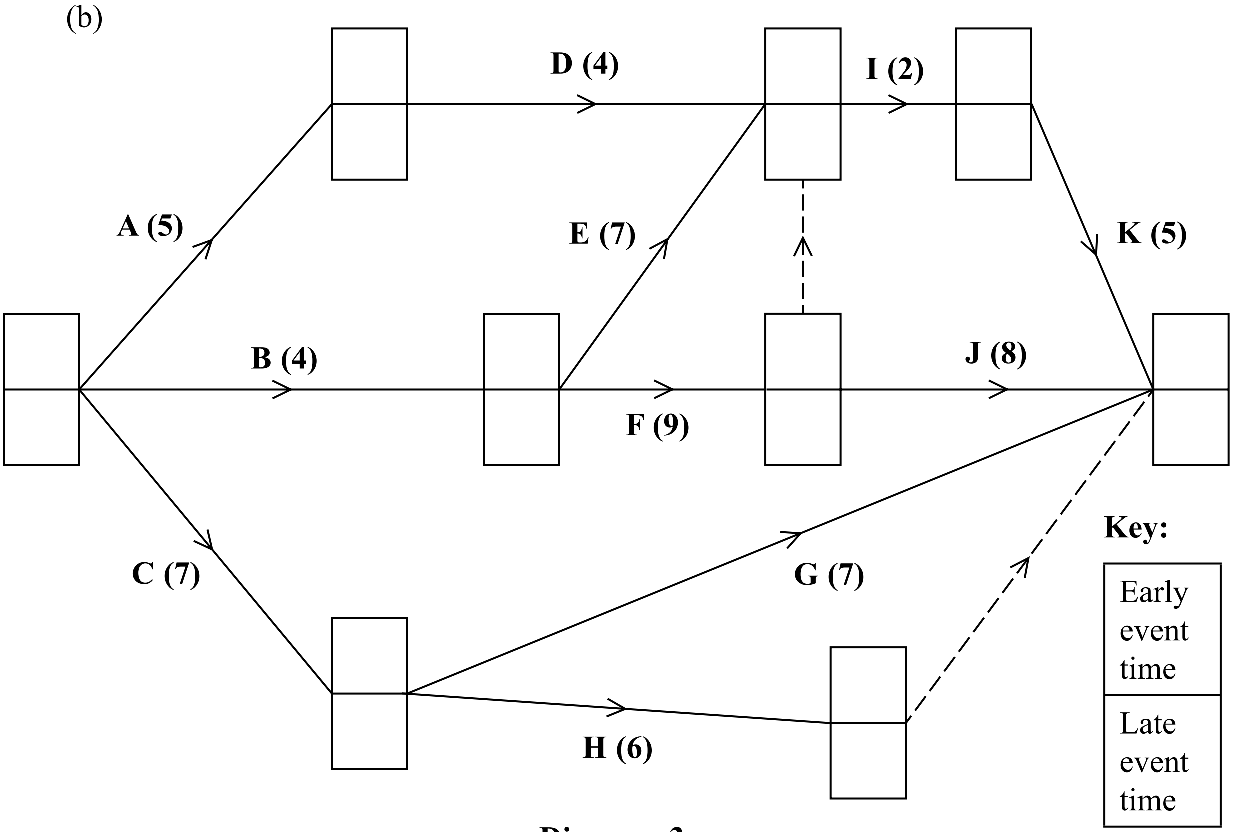
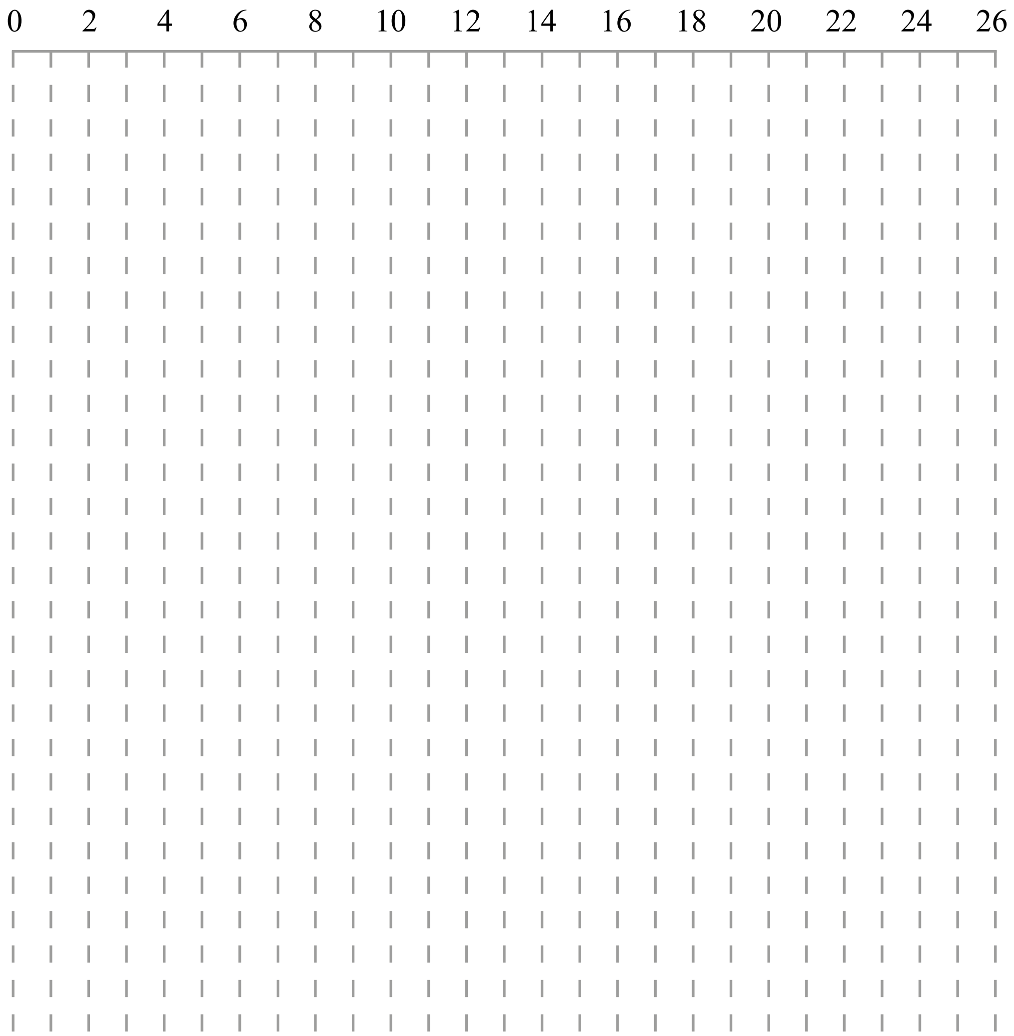


Diagram 3

Leave blank

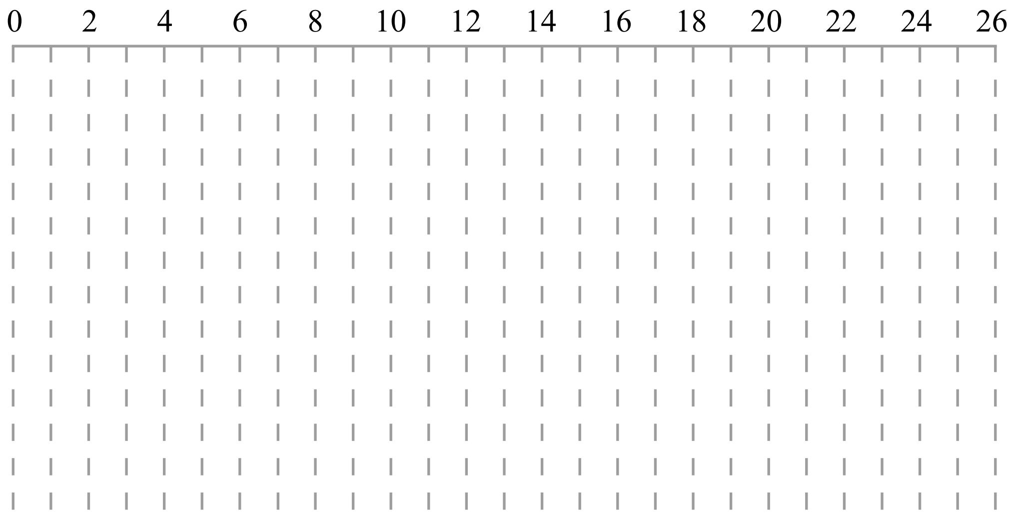
Question 6 continued

(e)



Grid 1

(f)



Grid 2

Q6

(Total for Question 6 is 16 marks)

TOTAL FOR PAPER IS 75 MARKS

Decision Mathematics D1 Mark scheme

Question	Scheme	Marks
1(a)	E.g. if use CD as shortcut get 807 or if use CF + AD get 793	M1 A1
		(2)
(b)	A F E D B C A	B1
	82 113 98 130 110 217 = 750	B1
		(2)
(c)	length of RMST = 439	B1
	439 + 82 + 113 = 634	M1 A1
		(3)
(d)	$634 < \text{optimal} \leq 750$	B1ft
		(1)
		(8 marks)

Notes:

(a)

M1: Their plausible shortcut leading to a value < 810 and a length below 810 stated.

A1: cao – shortcut and length must be consistent.
(Examples shortcuts: CD = 807, CF + AD = 793, CF + BD = 664, AD + EF + FC = 715, DF FC = 785 etc.)

(b)

B1: cao

B1: cao

(c)

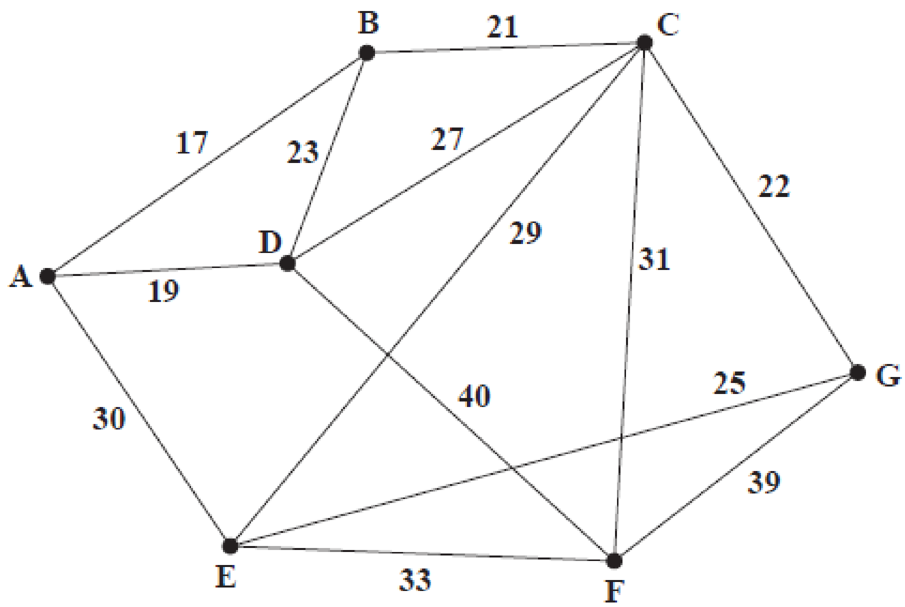
B1: cao

M1: Adding two least weighted arcs to their RMST length

A1: cao

(d)

B1: An interval that incorporates their lower bound from (c) and their best upper bound from either (a) or (b)

Question	Scheme	Marks
2(a)	e.g. accept (i) Every pair of nodes connected by a path (ii) Connected graph with no cycles (iii) All nodes connected	B1 B1 B1
		(3)
(b)	$n - 1$	B1
		(1)
(c)		M1 A1
		(2)
(d)	Kruskal: AB, AD, BC, CG, reject BD, EG, reject CD, reject CE, reject AE, CF	M1 A1 A1
		(3)
(e)	135 (km)	B1
		(1)
		(10 marks)

Notes:

(a)

In (a), all technical language used must be correct – for example, do not accept 'point' for node, etc

(i)B1: every pair and **path** (or clear definition of path) – no bod - not describing complete graph

(ii)B1: connected and **no cycles** (not 'loops', 'circles', etc. unless 'cycle' seen as well)

(iii)B1: all nodes connected (accept definition of minimum spanning tree)

(b)

B1: cao

(c)

M1: Either all five arcs correct (ignore weights) or at least three arcs correct (including weights)

A1: cso (arcs **and** weights) – no additional arcs

Question 2 notes *continued*	
(d)	
M1:	Kruskal's – first three arcs (AB, AD, BC,… or weights 17, 19, 21, …) chosen correctly **and at least one rejection seen at some point**. **For M1 only**: follow through from their diagram from (c)
A1:	All six arcs (AB, AD, BC, CG, EG, CF or weights 17, 19, 21, 22, 25, 31) chosen correctly and no additional arcs (no follow through from an incorrect network in (c))
A1:	cso All selections and rejections correct (in correct order and at the correct time) – do not accept weights or a contradiction between arcs and their weights (e.g. AB (16))
B1:	cao (ignore lack of units)

Question	Scheme	Marks
3(a)	Bin 1: <u>12.1</u> <u>9.3</u> <u>10.9</u> Bin 2: <u>15.7</u> $\boxed{6.4}$ $\boxed{7.9}$ Bin 3: $\boxed{17.4}$ 8.1 Bin 4: $\boxed{20.1}$ Bin 5: 14.0	<u>M1</u> $\boxed{\text{A1}}$ A1
		(3)
(b)	**(i)** 12.1 15.7 10.9 17.4 9.3 20.1 7.9 8.1 14.0 6.4 15.7 12.1 17.4 10.9 20.1 9.3 8.1 14.0 7.9 6.4	M1 A1
	(ii) Comparisons = 9 + 8 = 17 Swaps = 7 + 5 = 12	B1 B1
		(4)
(c)	e.g. middle right 12. 1 9.3 15.7 10.9 17.4 $\boxed{6.4}$ 20.1 7.9 8.1 14.0 Pivot 6.4 12.1 9.3 15.7 10.9 $\boxed{17.4}$ 20.1 7.9 8.1 14.0 <u>6.4</u> Pivot 17.4 20.1 <u>17.4</u> 12.1 9.3 15.7 $\boxed{10.9}$ 7.9 8.1 14.0 <u>6.4</u> Pivot (20.1) 10.9 20.1 <u>17.4</u> 12.1 $\boxed{15.7}$ 14.0 <u>10.9</u> 9.3 $\boxed{7.9}$ 8.1 <u>6.4</u> Pivots 15.7 7.9 20.1 <u>17.4</u> <u>15.7</u> 12.1 $\boxed{14.0}$ <u>10.9</u> 9.3 $\boxed{8.1}$ <u>7.9</u> <u>6.4</u> Pivots 14.0 8.1 20.1 <u>17.4</u> <u>15.7</u> <u>14.0</u> 12.1 <u>10.9</u> 9.3 <u>8.1</u> <u>7.9</u> <u>6.4</u> Sort complete	M1 (quick) A1 ($1^{st}/2^{nd}$ passes/pivot for 3^{rd}) A1ft ($3^{rd}/4^{th}$ passes/pivot for 5^{th}) A1(cso + 'sort complete')
		(4)
(d)	Bin 1: <u>20.1</u> $\boxed{12.1}$ Bin 2: <u>17.4</u> <u>14.0</u> Bin 3: <u>15.7</u> $\boxed{10.9}$ 6.4 Bin 4: $\boxed{9.3}$ $\boxed{8.1}$ 7.9	<u>M1</u> $\boxed{\text{A1}}$ A1
		(3)
(e)	e.g. $\frac{121.9}{33} \approx 3.694$ so yes 4 bins is optimal	B1ft
		(1)
		(15 marks)

Question 3 *continued*
Notes:

(a)

M1: First four numbers placed correctly (therefore Bin 1 correct and 15.7 in Bin 2) and at least seven numbers put in bins – condone cumulative totals here only

A1: First eight numbers placed correctly (therefore Bins 1 and 2 correct and 17.4 in Bin 3 and 20.1 in Bin 4)

A1: cso All correct

(b)

(i)M1: Bubble sort – first pass correct

(i)A1: cao both passes correct (ignore additional passes)

(ii)B1: cao on total number of comparisons

(ii)B1: cao on total number of swaps

SC in b(ii): If B0B0, award B1B0 if correct numbers referred to but not summed

(c)

M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – **choosing first/last item as pivot is M0**) and first pass gives $>p$, p, $<p$. So after the first pass the list should read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration M1 only**

A1: First and second passes correct **and** next pivot(s) chosen correctly for third pass (but third pass does not need to be correct)

A1ft: Third and fourth passes correct (follow through from their second pass and choice of pivots) – **and** next pivot(s) chosen correctly for the fifth pass

A1: cso (correct solution only – all previous marks in this part **must** have been awarded) including 'sort complete' – this could be shown by the final list being re-written or 'sorted' statement or each item being used (**not** just stated) as a pivot

(d)

M1: **Must be using 'sorted' list** in decreasing order (independent of (c)). First four numbers placed correctly and at least seven numbers put in bins – condone cumulative totals here only. First-fit increasing is M0

A1: First eight numbers placed correctly

A1: cso – all correct

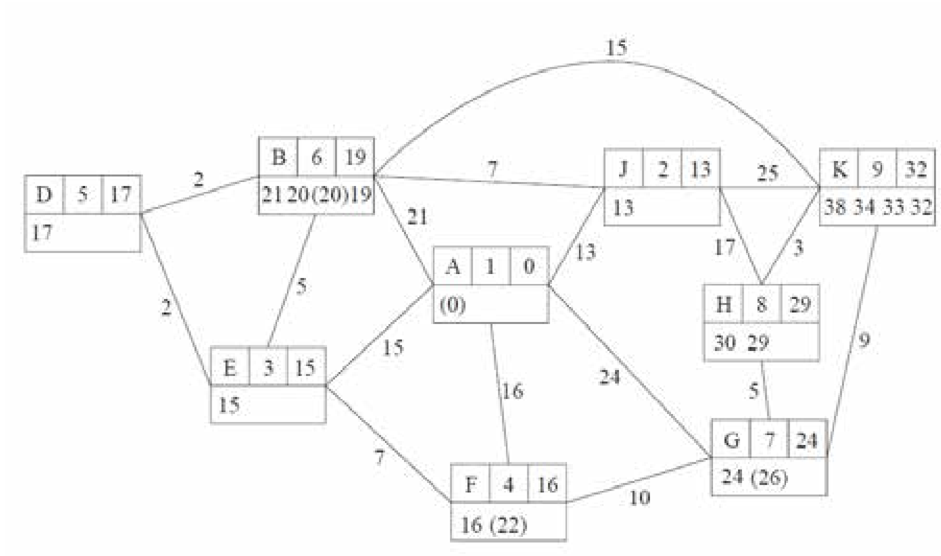
SC for (d): if the 'sorted' list they use in (d) has one 'error' from (c) (e.g. a missing number, an extra number or one number incorrectly placed) then M1 only can be awarded in (d) (for the first four numbers). If there is more than one 'error' then M0. Allow full marks in (d) if a correct list is used in (d) even if the list is incorrect at the end of (c).

(e)

B1ft: $\frac{121.9}{33}$ **or** awrt 3.7 (**or** 3.6 with correct calculation seen) **and** 4 together with a correct conclusion based on their answer to (d) (a correct calcuation etc. with an answer of 4 with no conclusion (as a minimum accept 'yes') scores B0)

middle left

12. 1 9.3 15.7 10.9 $\boxed{17.4}$ 6.4 20.1 7.9 8.1 14.0	Pivot 17.4
20.1 $\underline{17.4}$ 12.1 9.3 15.7 $\boxed{10.9}$ 6.4 7.9 8.1 14.0	Pivot (20.1) 10.9
20.1 $\underline{17.4}$ 12.1 $\boxed{15.7}$ 14.0 $\underline{10.9}$ 9.3 $\boxed{6.4}$ 7.9 8.1	Pivots 15.7 6.4
20.1 $\underline{17.4}$ $\underline{15.7}$ $\boxed{12.1}$ 14.0 $\underline{10.9}$ 9.3 $\boxed{7.9}$ 8.1 $\underline{6.4}$	Pivots 12.1 7.9
20.1 $\underline{17.4}$ $\underline{15.7}$ 14.0 $\underline{12.1}$ $\underline{10.9}$ $\boxed{9.3}$ 8.1 $\underline{7.9}$ $\underline{6.4}$	Pivot (14.0) 9.3
20.1 $\underline{17.4}$ $\underline{15.7}$ 14.0 $\underline{12.1}$ $\underline{10.9}$ $\underline{9.3}$ 8.1 $\underline{7.9}$ $\underline{6.4}$	(sort complete (8.1))

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4(a)</td><td>
</td><td>M1

A1
(JEFD)

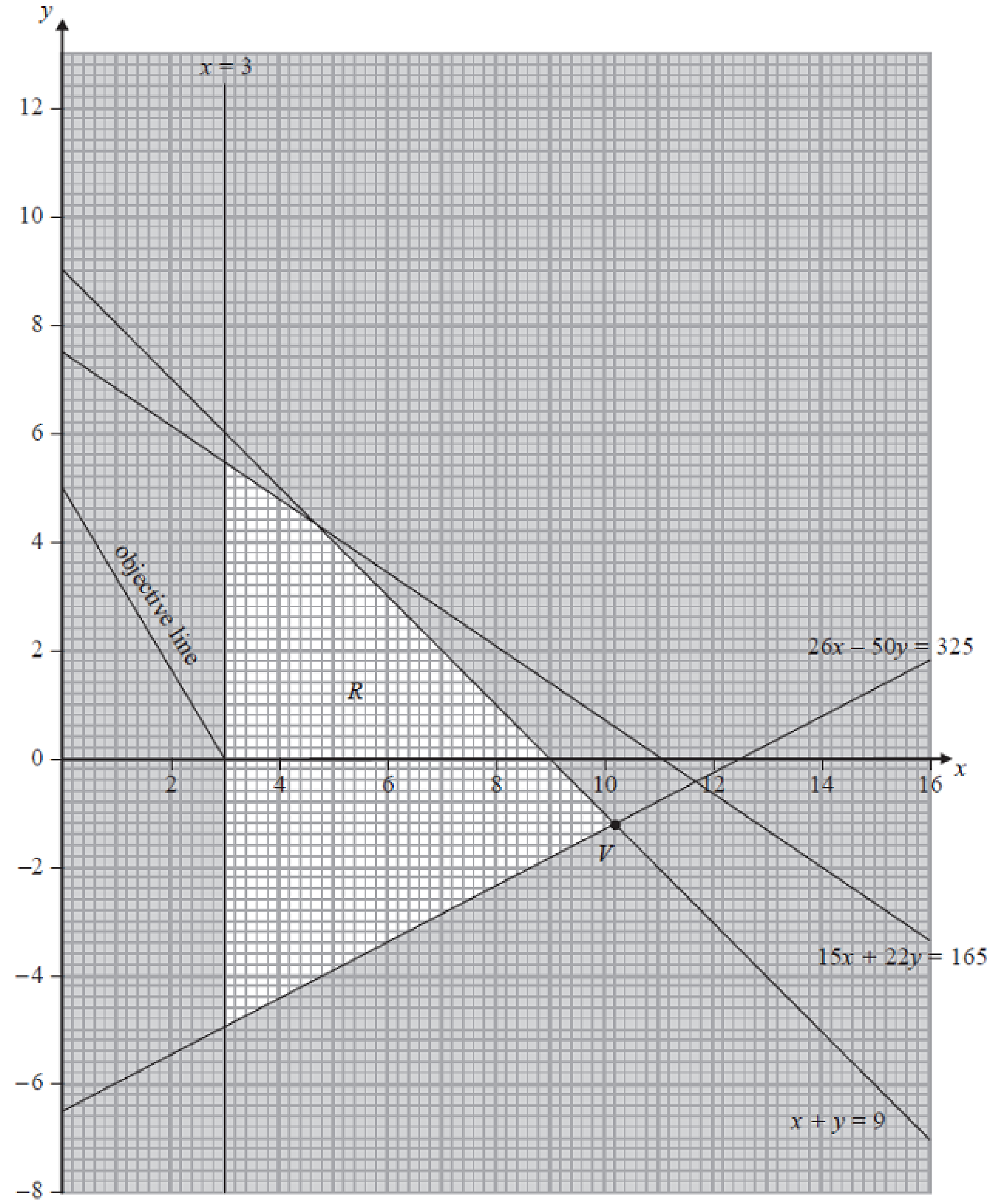
A1
(BG)

A1ft
(HK)</td></tr>
<tr><td></td><td>Quickest route: A – G – H – K</td><td>A1</td></tr>
<tr><td></td><td>Shortest time: 32 (mins)</td><td>A1ft</td></tr>
<tr><td></td><td></td><td>(6)</td></tr>
<tr><td>(b)</td><td>Route from B to K via A: B – D – E – A – G – H – K
Length: 51 (mins)</td><td>B1
B1ft</td></tr>
<tr><td></td><td></td><td>(2)</td></tr>
<tr><td>(c)</td><td>A(ED)B + F(G)H = 19 + 15 = 34
AF + B(K)H = 16 + 18 = 34
A(G)H + B(DE)F = 29 + 11 = 40

Arcs AF, BK, KH or AE, ED, DB, FG, GH will be traversed twice
Route length = 196 + 34 = 230 (mins)</td><td>M1
A1ft
A1ft
A1ft

A1A1
A1</td></tr>
<tr><td></td><td></td><td>(7)</td></tr>
<tr><td colspan="3">Notes:</td></tr>
<tr><td colspan="3">(a)
M1: A larger value replaced by a smaller value at least once in the working values at either B or H or K
A1: All values in J, E, F and D correct and the working values in the correct order. Penalise order of labelling only once per question. Condone an additional working value at F of 22
A1: All values in B and G correct and the working values in the correct order. Penalise order of labelling only once per question (B and G must be labelled in that order and B must be labelled after J, E, F, D). Condone an additional working value of 20 at B and an additional working value of 26 at G
A1ft: All values in H and K correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question (H and K must be labelled in that order and H labelled after all other nodes (excluding K))
A1: CAO (AGHK)
A1ft: Follow through on their final value at K – if their answer is not 32 follow through their final value at K (condone lack of units)</td></tr>
</table>

Question 4 notes *continued*	
(b)	
B1:	CAO (BDEAGHK)
B1ft:	51 or their final value at B + their final value at K (condone lack of units)
(c)	
M1:	Three distinct pairings of the correct four odd nodes
A1ft:	One row correct including pairing **and** total (the ft on the first three A marks in (c) is for using their final values at B, F and H from (a) for the lengths of AB, AF and AH only)
A1ft:	Two rows correct including pairing **and** totals
A1ft:	All three rows correct including pairing **and** totals
A1:	CAO one combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working)
A1:	CAO both combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working)
A1:	CAO (230)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>5(a)(b)</td><td>
</td><td>B1
B1
B1
B1 (R)
(4)
B1
B1
(2)</td></tr>
<tr><td>(c)</td><td>$V\left(\frac{775}{76},-\frac{91}{76}\right)$
$P=\frac{1801}{38}$</td><td>M1 A1
A1
(3)</td></tr>
<tr><td>(d)</td><td>$x = 3, y = -4$ minimum value is 3</td><td>B1 B1
(2)</td></tr>
<tr><td></td><td></td><td>(11 marks)</td></tr>
</table>

Question 5 *continued*

Notes:

(a)

In (a), lines must be long enough to define the correct feasible region **and** pass through one small square of the points stated:

$x + y = 9$ passes through (5, 4) and (9,0) but in most cases check (0, 9) and (9,0)

$26x - 50y = 325$ passes through (5, -3.9) and (10, -1.3) but in most cases check (0, -6.5) and (12.5, 0)

$15x + 22y = 165$ passes through $\left(3, \frac{60}{11}\right)$ and $\left(4, \frac{105}{22}\right)$ but in most cases check (0, 7.5) and (11, 0)

B1: Any two lines correctly drawn

B1: Any three lines correctly drawn

B1: All four lines correctly drawn

B1: Region, R, correctly labelled – not just implied by shading – dependent on scoring the first three marks in (a)

(b)

B1: Drawing the correct objective line on the graph, use line drawing tool to check if necessary. Line must not pass outside of a small square if extended from axis to axis

B1: V labelled clearly on their graph. **This mark is dependent on both the correct feasible region (but maybe not labelled) and the correct objective line**

(c)

M1: Candidates **must** have drawn either the correct objective line **or** its reciprocal. If they have drawn the correct objective line they must be solving $x + y = 9$ and $26x - 50y = 325$. If they have drawn the reciprocal objective line they must be solving $x = 3$ and $15x + 22y = 165$. Must get to either $x = \ldots$ or $y = \ldots$ (condone one error in the solving of the simultaneous equations). The correct exact answer $\left(\frac{775}{76}, -\frac{91}{76}\right)$, or for the reciprocal $\left(3, \frac{60}{11}\right)$, can imply this mark

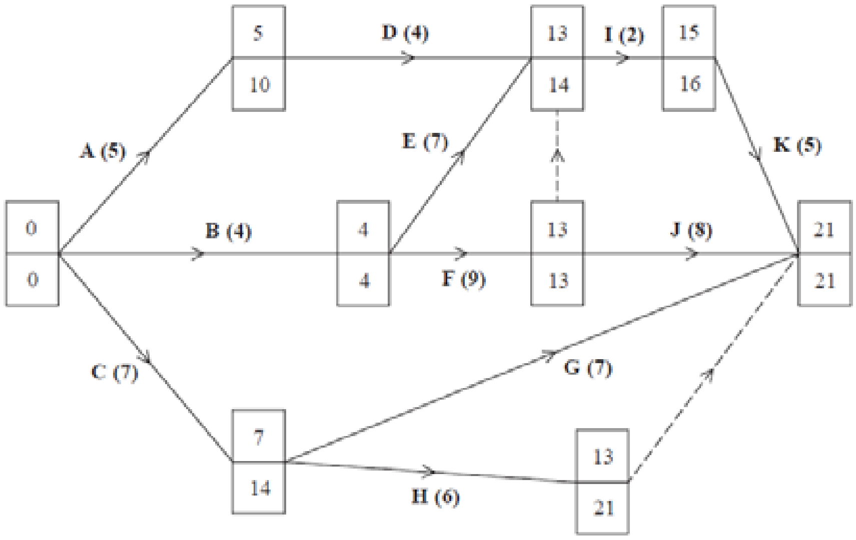
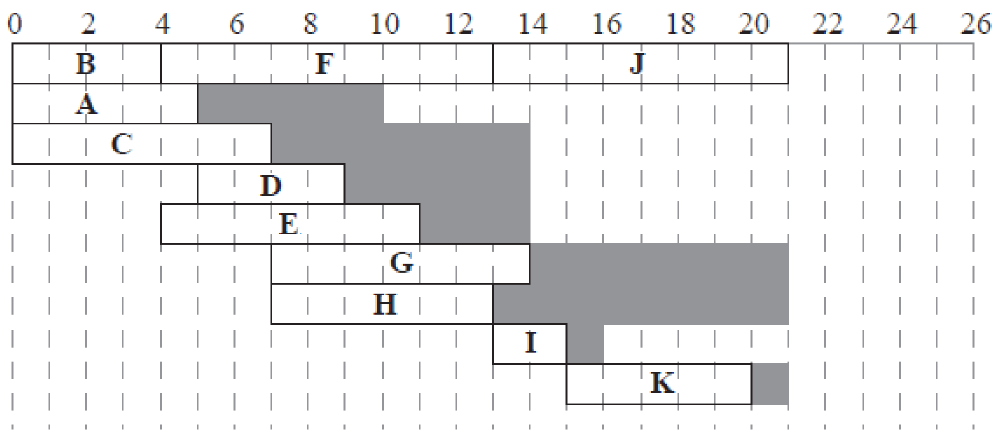
A1: cao $\left(\frac{775}{76}, -\frac{91}{76}\right)$ or $\left(10\frac{15}{76}, -1\frac{15}{76}\right)$ (coordinates must be exact) – **if correct answer stated with no working seen then award M1A0 only** (however, they can still earn the next A mark for the corresponding value of P at V). **This mark is dependent on the correct feasible region (but maybe not labelled)**

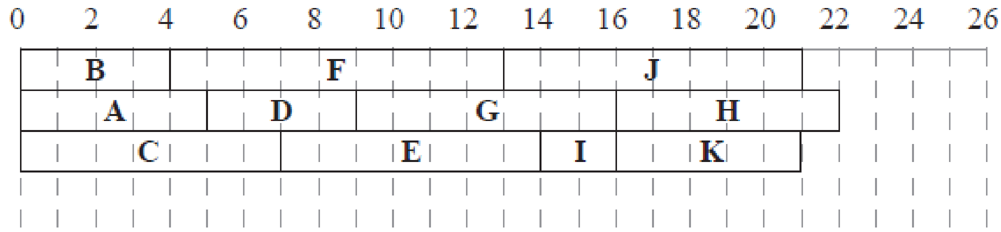
A1: cao $\frac{1801}{38}$ or $47\frac{15}{38}$ (must be exact). **This mark is dependent on the correct feasible region (but maybe not labelled)**

(d)

B1: cao $x = 3, y = -4$ or $(3, -4)$

B1: cao of 3

Question	Scheme	Marks
6(a)	**(i)** The dummy from event 5 to event 6 is needed to show that J depends on F but I depends on D, E and F	B1
	(ii) The dummy from event 7 to event 9 is because activities G and H must be able to be described uniquely in terms of the events at each end	B1
		(2)
(b)		M1 A1 M1 A1
		(4)
(c)	21 (hours)	B1
		(1)
(d)	$\frac{64}{21} \approx 3.048$ so at least 4 workers required	M1 A1
		(2)
(e)		M1 A1 M1 A1
		(4)

Question	Scheme	Marks
6(f)	e.g. 	M1 A1 A1
		(3)
		(16 marks)

Notes:

(a)

In (a) any use of the terms 'activity' and 'event' must be correct

B1: cao dependency - all relevant activities must be referred to - activities I, J, F and either D or E must be mentioned.

B1: cao uniqueness – please note that, for example, 'so that activities can be defined uniquely' is not sufficient to earn this mark. There must be some mention of describing activities in terms of the event at each end. However, give bod on statements that imply that an activity begins and ends at the same event

(b)

M1: **All** top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue

A1: cao (top boxes)

M1: **All** bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue

A1: cao (bottom boxes)

(c)

B1: cao (21)

(d)

M1: Attempt to find lower bound: (a value in the interval [55 – 73] / their finish time) **or** (sum of the activities / their finish time) **or** (as a minimum) an awrt 3.05 or 3.04 (truncated)

A1: cso – either a **correct** calculation seen **or** awrt 3.05 (or 3.04) **then** 4. An answer of 4 with no working scores M0A0

(e)

M1: At least 8 activities added including 5 floats. Scheduling diagram scores M0

A1: Critical activites dealt with correctly and 4 non-critical activities dealt with correctly

M1: All 11 activities including all 8 floats (on the correct non-critical activities)

A1: cao (all activities correct and present only once)

Question 4 notes *continued*

(f)

M1: Not a cascade chart. 3 workers used and at least 9 activities placed. The completion time must be no greater than one hour more than the minimum completion time stated in (c) or seen in (b)

A1: 3 workers, All 11 activities present (just once). Condone one error either precedence or activity length. The completion time must be one hour greater than the minimum completion time stated in (c) or seen in (b)

A1: 3 workers. All 11 activities present (just once). No errors. The completion time must be 22

Activity	Duration	IPA
A	5	-
B	4	-
C	7	-
D	4	A
E	7	B
F	9	B
G	7	C
H	6	C
I	2	D, E, F
J	8	F
K	5	I

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